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Physics
Mechanics

MECHANICS

PROBLEMS

FOR ENGINEERING STUDENTS

BY

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PREFACE

THERE is an opinion among engineers that too often students are not well grounded in the practical problems of Mechanics; that they know more of theory and mathematical deductions than of practical applications. A prominent educator has recently said to me, in regard to the teaching of Mechanics, "I am convinced that it is to be done more thoroughly in the future than in the past;" and it will be done, he believes, by sticking close to elementary principles as developed by well-chosen practical problems. Furthermore, he adds, "it will have to be recognized that all an engineering baccalaureate course can worthily accomplish is to give the raw recruit the 'setting-up' exercises in Mechanics."

It is now generally recognized, I think, that this subject should cover first of all the elements and fundamental principles that form the basis of every engineer's knowledge; that these necessary elements and principles are best understood and best remembered by actually solving numerous problems that present important facts illustrative of every-day engineering practice, and arouse the student's interest far better than abstract examples which can be easily formulated from imaginary conditions.

Therefore, for the reasons indicated above, an effort

has been made in preparing this book to present, from actual conditions, many practical problems together with brief definitions and solutions of typical problems which should help the student in Mechanics to follow the advice once given by George Stephenson to his son Robert :

> "Learn for yourself, think for yourself,
> make yourself master of principles."

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FRANK B. SANBORN.

TUFTS COLLEGE, MASS.,
June, 1906.

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MECHANICS–PROBLEMS.

INTRODUCTION.

The problems and solutions that follow have been arranged in the order of Work, Force, and Motion. At the beginning of each important section one problem has been solved so as to explain the method of solving similar problems and to serve as a guide for solutions to be put in note-books. An effort has been made throughout the book to simplify. Few methods have been presented; the calculus has been used only where necessary; no discussion has been offered of the term mass — many such subjects have been left for more advanced courses or extended treatises.

The "gravitation system" of units — the foot-pound–second system, or meter-kilogram–second system — known as the engineers' system has been used exclusively.

In engineering practice one is often puzzled to tell just what data to collect and afterward how much of it to use; because of this, I have left more data in some of the problems, and especially those under Review, than is absolutely necessary for solving the problem, and the student will have opportunity "to pick and choose" just as he would do in actual cases.

DEFINITIONS.

Mechanics is the science that treats of the action of forces at rest and in motion.

Work, Force, and **Motion** are the three sub-divisions of Mechanics considered in this book.

WORK.

Work is done by the action of force through some distance.

Work is measured by the product of force times the distance through which it acts.

Work = force × distance, — a formula fundamental for all Work problems.

Energy is the amount of work that a body possesses.

Potential energy is the work that a body possesses by virtue of its position above the earth's surface.

Kinetic energy is the work that a body possesses by virtue of its velocity.

Horse-power is the rate of doing work. One horse-power is the equivalent of 33 000 foot-pounds of work done per minute.

FORCE.

Force in Mechanics has both magnitude and direction, and in this treatise

Force Magnitude is usually expressed in pounds. It may act as pressure, a push, or as tension, a pull.

Concurrent forces acting on a body are those that have the same point of application.

Non-concurrent have different points of application.

Moment of a force about a point or axis is the product obtained by multiplying the magnitude of the force by the shortest distance from the point or axis to the line of action of the force.

Moment = force × perpendicular. Clockwise tendency of rotation is usually taken positive.

Resultant of a system of concurrent forces is a single force that might be substituted for them without changing the effect.

Equilibriant of a system of forces is a single force that balances them. The equilibriant is equal and opposite to the resultant.

Components of a single force are the forces that might be substituted for it without changing the effect.

Parallelogram of forces. When three forces that are in equilibrium meet in a point they can be represented in magnitude and direction by a diagonal and the sides of a parallelogram. This parallelogram is called the parallelogram of forces.

1. **Σ Vertical components = 0.** When the forces acting in one plane upon a body are in equilibrium, the forces can be resolved into components in any one direction, and the algebraic sum of the components will equal 0. Likewise,

2. **Σ Horizontal components = 0.** The algebraic sum of the components in a direction perpendicular to that of 1 will equal zero; and

3. **Σ Moments = 0.** The algebraic sum of the moments of the forces taken about any point or axis in the plane will equal zero.

These three axioms can frequently be used to formulate three equations that contain unknown quantities which can then be determined.

A Couple consists of two equal, opposite, parallel forces not acting in the same straight line.

Moment of a couple is the product of one of the equal forces by the perpendicular distance between them.

Center of gravity of a body or a system of bodies is a point about which the body or system can be imagined to balance and the forces of gravity will cause no rotation.

Centroid and **Center of Mass** are terms that are sometimes used in preference to center of gravity.

Centroid is the point of application of a system of parallel forces.

MOTION.

Motion (uniform) is that in which a body moves through equal distances in equal times.

Motion (accelerated) is that in which a body moves through unequal distances in equal times.

Motion (uniform-accelerated) is that in which the velocity increases the same amount in each unit of time, which is generally taken as the second.

Acceleration is the gain or loss in velocity per unit of time.

Centrifugal force. When a body is compelled to move in a curved path it exerts a force directed outwards from the center; its amount is the centrifugal force $= \frac{W}{g}\frac{v^2}{r}$.

Impact is said to take place when one body strikes against another.

A period of compression thus occurs, and the forces acting are **Impulsive forces** of compression. Then follows **a period of restitution.**

Coefficient of restitution e for any pair of substances is the ratio of the impulsive force of restitution to the impulsive force of compression.

WORK. Up grade on the Pennsylvania Railroad at Tyrone, Penn.

I. WORK. — FOOT-POUNDS.

1. A 20th-century express having 5 parlor cars each of 75 tons weight, a locomotive of 105 tons, and a tender of 60 tons goes up a grade of 1 vertical in 120 horizontal; the resistances are 15 pounds per ton. Find the amount of work that locomotive expends per mile of travel.

$$\begin{aligned}
\text{Work}_{\text{of locomotive}} &= \text{work}_{\text{of friction}} + \text{work}_{\text{of lifting train}} \\
\text{Work}_{\text{of friction}} &= \text{force} \times \text{distance} \\
\text{Force} &= 15 \times 540 \\
&= 8\ 100 \text{ pounds} \\
\text{Distance} &= 1 \text{ mile} \\
&= 5\ 280 \text{ feet} \\
\therefore \text{Work}_{\text{of friction}} &= 8\ 100 \times 5\ 280 \\
&= 42\ 768\ 000 \text{ foot-pounds} \\
\text{Work}_{\text{of lifting train}} &= \text{force} \times \text{distance} \\
\text{Force} &= 540 \times 2\ 000 \\
&= 1\ 080\ 000 \text{ pounds} \\
\text{Distance} &= 5\ 280 \times \tfrac{1}{120} \\
&= 44 \text{ feet} \\
\therefore \text{Work}_{\text{of lifting train}} &= 47\ 520\ 000 \text{ foot-pounds} \\
\text{Work}_{\text{of locomotive}} &= 42\ 768\ 000 + 47\ 520\ 000 \\
&= 90\ 288\ 000 \text{ foot-pounds}
\end{aligned}$$

2. Find the work done by a locomotive in drawing a train 1 mile along a level track when the constant resistances of friction, air, and so on are 1 ton.

3. A punch exerts a uniform pressure of 36 tons in punching a hole through an iron plate of one-half inch thickness. Find the foot-pounds of work done.

4. Find what work is being done per minute by an engine that is raising 2 000 gallons of water an hour from a mine 300 feet deep.

5. If a weight of 1 130 pounds be lifted up 20 feet by 20 men twice in a minute, how much work does each man do per hour?

6. A number of men can each do, on the average, 495 000 foot-pounds of work per day of 8 hours. How many such men are required to do 33 000 × 10 foot-pounds of work per minute?

7. A centrifugal pump delivers water 10 feet above the level of a lake of half a square mile area. At the end of a day's pumping, the water has been lowered $1\frac{1}{2}$ feet. How much work has been done?

"Distance" will be 10 feet plus $\frac{1}{2}$ of $1\frac{1}{2}$ feet.

8. Water in a well is 20 feet below the surface of the ground, and when 500 gallons have been pumped out it is 26 feet below. Find the work done.

9. Brick and mortar for a chimney 100 feet high are raised to an average height of 35 feet. Total amount of material used 40 000 cubic feet or about 5 600 000 pounds. What work was done?

10. What work is done in winding up a chain that hangs vertically, is 130 feet long, and weighs 20 pounds per foot?

11. A chain of weight 300 pounds and length 150 feet, with a weight of 500 pounds at the end of it, is wound up by a capstan. What work is done?

12. A stream of width 20 feet, average depth 3 feet, and mean velocity of 3 miles per hour has an available fall of 80 feet. What work is stored in the quantity of water flowing each minute?

Find the pounds of water flowing by observing that
Quantity = area × velocity.

13. A horse draws 150 pounds of earth out of a well, by means of a rope going over a fixed pulley, which moves at the rate of $2\frac{1}{2}$ miles an hour. Neglecting friction, how many units of work does this horse perform a minute?

14. A cylindrical shaft 14 feet in diameter must be sunk to a depth of 10 fathoms through chalk, the weight of which is 144 pounds per cubic foot. Find the work done in raising the chalk.

15. A well is to be dug 20 feet deep and 4 feet in diameter. Find the work in raising the material, supposing that a cubic foot of it weighs 140 pounds.

16. A horse draws earth from a trench by means of a rope going over a pulley. He pulls up, twice every 5 minutes, a man weighing 130 pounds, and a barrowful of earth weighing 260 pounds. Each time the horse goes forward 40 feet. Find the useful work done per hour.

17. A body weighing 50 pounds slides a distance

of 8 feet down a plane inclined 20° to the horizontal, against a constant retarding force of 4 pounds. Compute the total work done upon the body by (gravity) its weight and the friction.

18. What work is stored in a cross-bow whose cord has been pulled 15 inches with a maximum force of 224 pounds?

19. If 25 cubic feet of water are pumped every 5 minutes from a mine 140 fathoms deep, what amount of work is expended per minute?

20. In pumping 1 000 gallons from a water-cistern with vertical sides the surface of the water is lowered 5 feet. Find the work done, the discharge being 10 feet above the original surface.

21. A uniform beam weighs 1 000 pounds, and is 20 feet long, it hangs by one end, round which it can turn freely. How many foot-pounds of work must be done to raise it from its lowest to its highest position?

22. A body is suspended by an elastic string of unstretched length 4 feet. Under a pull of 10 pounds the string stretches to a length of 5 feet. Required the work done on the body by the tension of the string while its length changes from 6 feet to 4 feet.

23. A weight of 200 pounds is to be raised to a height of 40 feet by a cord passing over a fixed smooth pulley; it is found that a constant force P, pulling the cord at its other end for three-fourths of

the ascent, communicates sufficient velocity to the weight to enable it to reach the required height. Find P.

$$\text{Work} = \text{force} \times \text{distance}$$

$$\text{Work}_{\text{on weight}} = 200 \times 40$$

$$\text{Work}_{\text{by pull}} = P \times \tfrac{3}{4} \text{ of } 40$$

$$\text{Work}_{\text{on weight}} = \text{Work}_{\text{by pull}}$$

$$200 \times 40 = P \times 30$$

$$P = 266\tfrac{2}{3} \text{ pounds}$$

24. A horse drawing a cart along a level road at the rate of 2 miles per hour performs 29 216 foot-pounds of work in 3 minutes. What pull in pounds does the horse exert in drawing the cart?

25. It is said that a horse can do about 13 200 000 foot-pounds of work in a day of 8 hours, walking at the rate of $2\frac{1}{2}$ miles per hour. What pull in pounds could such a horse exert continuously during the working-day?

26. If a horse walking once round a circle 10 yards across raises a ton weight 18 inches, what force does he exert over and above that necessary to overcome friction?

27. A building of weight 50 000 pounds is being moved on rollers by a horse that is pulling on a pole with a distance of 10 feet from the center of a capstan that is 18 inches in diameter. If the total friction is 200 pounds per ton, what force must the horse exert?

28. The 500-pound hammer of a pile-driver is raised to a height of 20 feet and then allowed to fall upon the head of a pile, which is driven into the ground 1 inch by the blow. Find the average force which the hammer exerts upon the head of the pile.

$$\begin{aligned} \text{Work} &= \text{force} \times \text{distance} \\ &= 500 \times 20 \\ &= 10\,000 \text{ foot-pounds} \\ \text{Distance} &= \tfrac{1}{12} \text{ foot} \\ \therefore 10\,000 \text{ foot-pounds} &= \text{force} \times \tfrac{1}{12} \text{ foot} \\ \therefore \text{force} &= 10\,000 \times 12 \\ &= 120\,000 \text{ pounds} \end{aligned}$$

29. A hammer weighing 1 ton falls from a height of 24 feet on the end of a vertical pile, and drives it half an inch deeper into the ground. Assume the driving force of the hammer on the pile to be constant while it lasts, and find its amount expressed in tons weight.

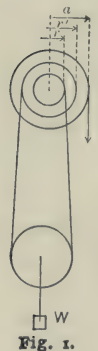


Fig. 1.

30. Determine by the principle of work, neglecting friction, the relation between the pull P and the load W in case of the differential wheel-and-axle of Fig. 1.

For one revolution,

$$\begin{aligned} \text{Work}_{\text{of P}} &= P \times 2\pi a \\ \text{Work}_{\text{on weight}} &= \tfrac{1}{2} W \times 2\pi r' - \tfrac{1}{2} W \times 2\pi r \\ P \times 2\pi a &= \tfrac{1}{2} W\pi \times 2 (r' - r) \\ P \times 2a &= W (r' - r) \\ \frac{P}{W} &= \frac{r' - r}{2a} . \end{aligned}$$

31. A barrel of Portland cement that weighs 396 pounds is to be hoisted by a wheel and axle as in Fig. 1; the radii are 6, 12 and 18 inches. What force will be required?

32. If, neglecting frictions, a power of 10 pounds, acting on an arm 2 feet long, produces in a screw-press a pressure of half a ton, what would be the pitch of the screw?

33. What is the ratio of the weight to the power, in a screw-press working without friction, when the screw makes 4 turns in the inch, and the arm to which the power is applied is 2 feet long?

34. What force applied at the end of an arm 18 inches long will produce a pressure of 1 000 pounds upon the head of a smooth screw when 11 turns cause the head to advance two-thirds of an inch?

35. Find the mechanical advantage in a differential screw, if the length of the power arm is 2 feet, and there are 4 threads to the inch in the large screw, and 5 threads to the inch in the small screw.

36. In a differential pulley, if the radii of the pulleys in the fixed block are as 3 to 2; and if the weight of the lower block is $1\frac{1}{2}$ pounds, what weight can be raised by a force of 5 pounds?

37. In a wheel and axle the diameter of the wheel is 7 feet, of the axle 7 inches. What weight can be

raised by a force of 10 pounds acting at the circumference of the wheel?

38. A weight of 448 pounds is raised by a cord which passes round a drum 3 feet in diameter, having on its shaft a toothed wheel also 3 feet in diameter; a pinion 8 inches in diameter, and driven by a winch handle 16 inches long, gears with the wheel. Find the power to be applied to the winch handle in order to raise the weight.

Fig. 2.

39. A tackle is formed of two blocks, each weighing 15 pounds, the lower one being a single movable pulley, and the upper or fixed block having two sheaves; the parts of the cord are vertical, and the standing end is fixed to the movable block. What pull on the cord will support 200 pounds hung from the movable block? and what will then be the pull on the staple at the upper block?

40. A weight of 400 pounds is being raised by a pair of pulley blocks, each having two sheaves; the standing part of the rope is fixed to the upper block, and the parts of the rope, whose weight may be disregarded, are considered to be vertical; each block weighs 10 pounds. What is the pressure on the point from which the upper block hangs?

41. Two equal weights, each 112 pounds, are joined by a rope which runs over two pulleys A and B 12

feet apart and in the same horizontal line. If a weight of ten pounds is lowered on to the rope halfway between A and B how far will the rope deflect?

Work of 10-pound weight = Work of two 112-pound weights.

42. A weight of 500 pounds, by falling through 36 feet, lifts, by means of a machine, a weight of 60 pounds to a height of 200 feet. How many units of work has been expended on friction, and what ratio does it bear to the whole amount of work done?

43. The pull on a tram-car was registered when the car was at the following distances along the track; 0 feet, 200 pounds; 10 feet, 150 pounds; 25 feet, 160 pounds; 32 feet, 156 pounds; 41 feet, 163 pounds; 56 feet, 170 pounds; 60 feet, 165 pounds; 73 feet, 160 pounds. What effective work was done in pulling the car through the distance of 73 feet, and what constant pull would have produced the same work?

44. In lifting an anchor of $1\frac{1}{2}$ tons from a depth of 15 fathoms in 6 minutes, what is the useful man-power, if a man-power is defined as 3 500 foot-pounds per minute?

45. Four hundred weight of material are drawn from a depth of 80 fathoms by a rope weighing 1.15 pounds per linear foot. How much work is done altogether, and how much per cent is done in lifting the rope? How many units of 33 000 foot-pounds per minute would be required to raise the material in $4\frac{1}{2}$ minutes?

HORSE-POWER

46. A gas engine must hoist 3 tons of grain through a vertical height of 50 feet every minute. What horse-power must be provided?

$$\text{Work}_{\text{of engine}} = \text{force} \times \text{distance}$$

$$= (3 \times 2\,000) \text{ pounds} \times 50 \text{ feet per [minute}$$

Now 1 horse-power = 33 000 foot-pounds per minute

$$\therefore \text{Work}_{\text{of engine}} = \frac{3 \times 2\,000 \times 50}{33\,000}$$

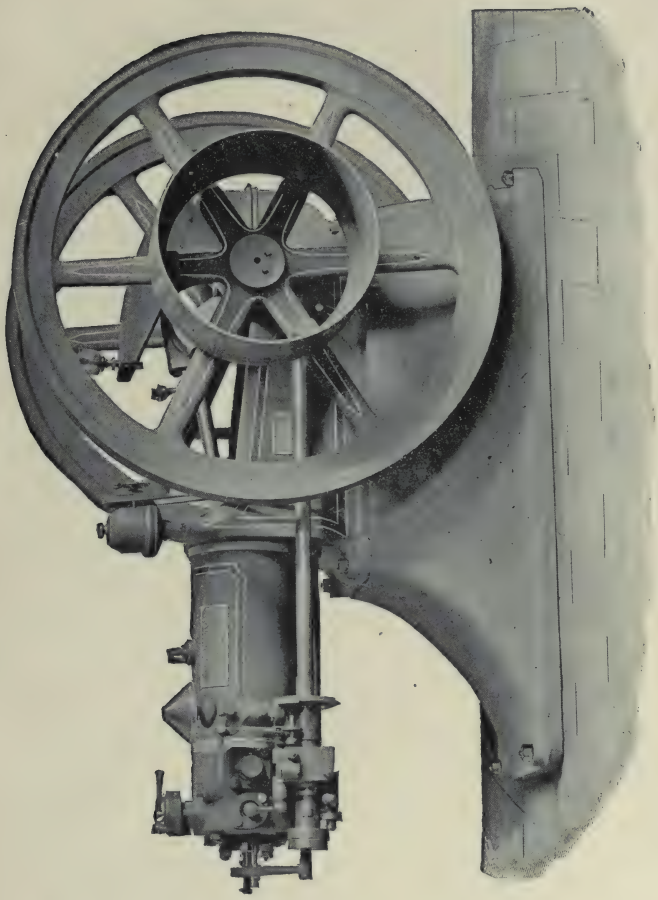
$$= 9\tfrac{1}{11} \text{ horse-power}$$

47. A hod-carrier who weighs 155 pounds carries 65 pounds of brick to the third story, a vertical height of 20 feet. How many foot-pounds of work has he done? If he makes 10 such trips in an hour, at what rate in horse-power does he work?

48. A windmill raises by means of a pump 22 tons of water per hour to a height of 60 feet. Supposing it to work uniformly, calculate its horse-power.

49. The travel of the table of a planing-machine which cuts both ways is 9 feet. If the resistance while cutting be taken at 400 pounds, and the number of revolutions or double strokes per hour be 80, find the horse-power absorbed in cutting.

50. A forge hammer weighing 300 pounds makes 100 lifts a minute; the perpendicular height of each lift is 2 feet. What is the horse-power of the engine that operates 20 such hammers?



51. An Otto gas engine is shown in the above illustration. It has a belt pulley that is 36 inches in diameter, and makes 150 revolutions per minute.

What force for driving, shafting, and machinery, therefore, can the belt transmit when the engine is developing its rated horse-power of twenty-one?

52. How many horse-power would it take to raise 3 hundred weight of coal a minute from a pit whose depth is 660 feet?

53. Find the horse-power of an engine which is to raise 30 cubic feet of water per minute from a depth of 440 feet.

54. Find the horse-power required to draw a train of 100 tons, at the rate of 30 miles an hour, along a level railroad, the resistance from friction being 16 pounds per ton.

55. Each of the two cylinders in a locomotive engine is 16 inches in diameter and the length of crank is 1 foot. If the driving-wheels make 105 revolutions per minute, and the mean effective steam-pressure is 85 pounds per square inch, what is the horse-power?

56. The weight of a train is 95.5 tons, and the drawbar pull is 6 pounds per ton. Find the horse-power required to keep the train running at 25 miles per hour.

57. A train, whose weight including the engine is 100 tons, is drawn by an engine of 150 horse-power; friction is 14 pounds per ton — all other resistances neglected. Find the maximum speed which the engine is capable of maintaining on a level track.

In the electrical problems that follow observe that

1 kilowatt = 1.340 horse-power

1 horse-power = 746 watts

Watts = volts × ampères

58. A dynamo is driven by an engine that develops 230 horse-power. If the efficiency of dynamo is 0.81 what "activity" in kilowatts is represented by the current generated?

59. Electric lamps giving 1 candle-power for 4 watts (*a*) how many 10- and (*b*) how many 16-candle lamps may be worked per electric horse-power? The combined efficiency of engine, dynamo, and gearing being 70 per cent, what is the candle-power available for every indicated horse-power?

60. What electrical current expressed in ampères will be used by a 250-volt electric hoist when raising 2 500 pounds of coal per minute from a ship's hold 150 feet below dump cars on trestle work, the efficiency of the whole arrangement being 50 per cent?

61. A prospective electric company can find a market for 900 electrical horse-power at a city 20 miles from a suitable water-power. Engineers estimate losses in generating machinery 10%; in line 7%; in transformers at load end 10%; and the efficiency of turbines 85%. The average velocity of the river is 2 feet per second; width available near dam 40 feet; depth 5 feet. Find (a) the water-power that would be required (b) the net fall that proposed dam must afford.

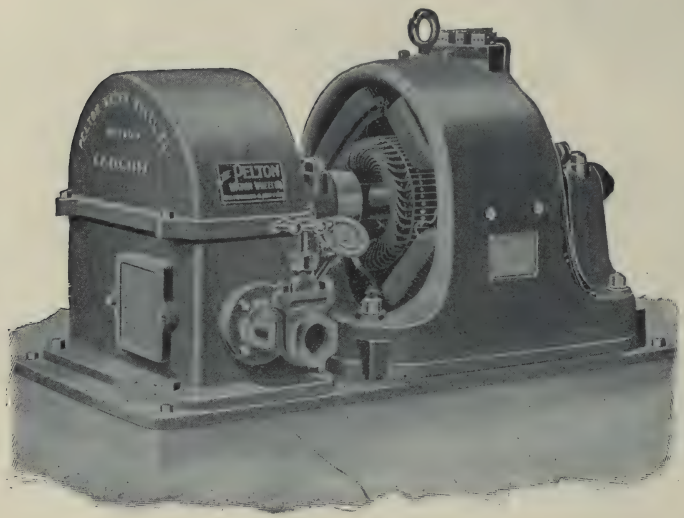


Fig. 3.

62. A water-motor is driven by two jets 1 inch in diameter, flowing with velocity of 80 feet per second. Theoretic horse-power would be 9.9; and if efficiency of wheel is 85 per cent, and the generator which the wheel drives also 85 per cent, what power in kilowatts does the current represent?

63. What is the difference in tensions of the two sides of a 30-inch driving belt that is running 4 200 feet a minute, and transmitting 300 horse-power?

In belt problems the difference in tensions represents "force."

64. Find the speed of a driving-pulley 3.5 feet in diameter to transmit 6 horse-power, the driving-force of the belt being 150 pounds.

65. A belt is designed to stand a difference in tension of 100 pounds only. Find the least speed at which it can be driven to transmit 20 horse-power.

66. A pulley 3 feet 6 inches in diameter, and making 150 revolutions a minute, drives by means of a belt, a machine which absorbs 7 horse-power. What must be the width of the belt so that its greatest tension may be 70 pounds per inch of width, it being assumed that the tension in the driving-side is twice that on the slack side?

67. An endless cord stretched and running over grooved pulleys with a linear velocity of 3 000 feet per minute, transmits 5 horse-power. Find the difference in tensions of the cord in pounds.

68. A rope drive has a grooved pulley 14 feet in diameter that makes 30 revolutions per minute. The difference in tensions being 100 pounds, find the horse-power transmitted.

69. A locomotive that can develop 1 000 horse-power is drawing a train of total weight 600 tons up a 2 per cent grade; resistances are 10 pounds per ton. Find the highest speed that can be attained.

$$\begin{array}{l} \text{Work of locomotive} = \text{Work of resistance} + \text{Work of lifting train} \\ 1\,000 \times 33\,000 = 10 \times 600 \times d + 600 \times 2\,000 \times \frac{2}{100} \times d \\ 33\,000 = 6 \times d + 6 \times 2 \times 2 \times d \\ 30\,d = 33\,000 \\ d = 1\,100 \text{ feet per minute} \\ = 12\tfrac{1}{2} \text{ miles per hour.} \end{array}$$

70. A train of 100 tons weight runs at 42 miles an hour on a level track; resistances are 8 pounds per ton. Find the speed of train up a 1 per cent grade (1 foot rise in 100 feet horizontal) if the engine-power is kept constant.

71. In 1895 a passenger engine on the Lake Shore Railroad made a run of 86 miles at the rate of 73 miles an hour. Weight of train, 250 tons; resistance on level track, 15 pounds per ton. The engine was a 10-wheeler, having drivers 5 feet 8 inches in diameter and cylinders 17×24 inches. When 730 horse-power was developed up a 1 per cent grade what was the average draw-bar pull?

72. A 98-horse-power automobile has by test in Colorado drawn a special 36-ton locomotive up a 12 per cent highway grade at the rate of four miles an hour. What were the frictional resistances per ton?

73. A modern farming machine equipped with a 100-horse-power automobile will plow, sow, and harrow, all at the same time, a strip 30 feet wide at the rate of $3\frac{1}{2}$ miles an hour, or 80 acres a day. What force is developed for each foot width of ground?

74. Find the total horse-power of two engines which are taking a train of 250 tons down a grade of 1 in 200 at 60 miles an hour, supposing the resistance on the level at this speed to be 35 pounds a ton.

75. An automobile that weighs 5 tons goes up a rough road of grade 1 vertical to 10 horizontal; air and frictional resistances are 16 pounds per ton. What horse-power must the motor develop to maintain a speed of 20 miles an hour?

76. Find the horse-power of a locomotive which is to move at the rate of 20 miles an hour up an incline which rises 1 foot in 100, the weight of the locomotive and load being 60 tons, and the resistance from friction 12 pounds per ton.

77. A steam-crane, working at 3 horse-power, is able to raise a weight of 10 tons to a height of 50 feet in 20 minutes. What part of the work is done against friction? If the crane is kept at similar work for 8 hours, how many foot-pounds of work are wasted on friction?

78. The six-master shown on the next page carries 5 500 tons of coal. It is unloaded by small engines which take up 1 ton at each hoist; average lift from hold of ship to top of chutes which lead to cars, 35 feet; weight of bucket, 1 ton; 2 trips are made per minute, and 25 per cent of power of engine is lost in friction and transmission. When two towers are working how long will it take to unload the vessel?

Six-masted Schooner Unloading Coal.

The illustration of six-master on opposite page accompanies Problem 78.

79. An average size coal barge will carry 1 600 tons. If it is unloaded by two simple direct engines, the coal being hoisted 65 feet to an elevated hopper on the wharf, weight of bucket 1 ton, and carrying 1 ton of coal, what horse-power of engines would be required to unload the 1 600 tons in 20 hours?

80. The locomotive of problem 71 made 360.7 revolutions per minute. What was the mean effective cylinder pressure?

$$\text{Work} = \text{force} \times \text{distance}$$

$$\text{Force} = \tfrac{1}{4}\pi\, 17^2 \times P \times 2$$

$$\text{Distance} = 2 \times 360.7 \times \tfrac{24}{12}$$

$$730 \times 33\,000 = (\tfrac{1}{4}\pi\, 17^2 \times P \times 2) \times (2 \times 360.7 \times \tfrac{24}{12})$$

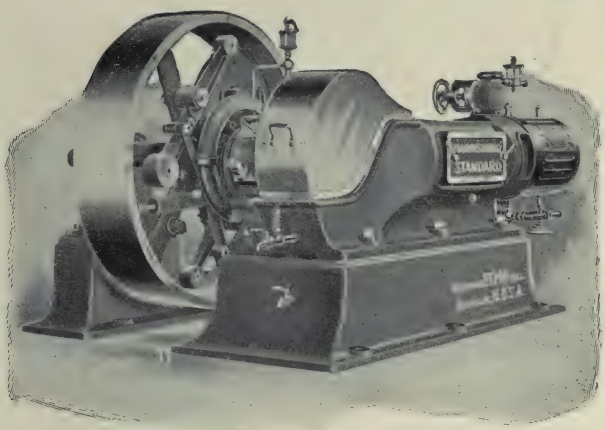


Fig. 4.

81. The engine shown in Fig. 4 has steam cylinder 15 inches in diameter; length of stroke, 15 inches;

revolutions per minute, 275; mean effective pressure, 38 pounds per square inch. Find the horse-power.

82. The indicator cards illustrated herewith were taken from an engine of the type shown in problem 81, diameter of steam cylinder being 14 inches, length of stroke 12 inches, revolutions per minute 300. Scale on cut the mean ordinates, which were produced by indicator springs of stiffness 40 pounds to an inch, and compute the indicated horse-power of the engine.

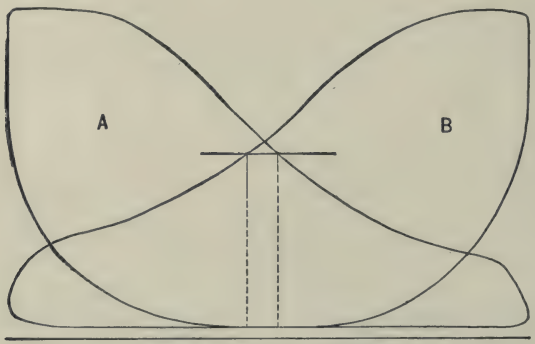


Fig. 5. Full Load Indication.

83. The indicator cards shown below were taken from one of the triple-expansion pumping-engines at the East Boston Station of the Metropolitan Sewerage. The cards were from two ends of a high-pressure cylinder. Refer to the cards and compute the indicated horse-power. (A twenty-four hours' duty trial of this pumping-engine was made January 17–18, 1901, by engineering students of Tufts College.)

Fig. 6. Head end. Card shown, one-half size; area of original, 4.69 square inches; stiffness of spring, 50 pounds per square inch; length of stroke, 30 inches; revolutions per minute, 84; piston diameter, 13½ inches.

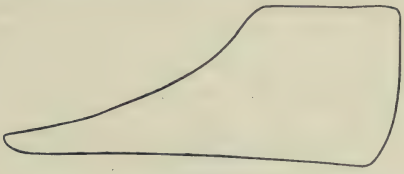


Fig. 7. Crank end. Card shown, one-half size; area of original, 4.62 square inches: stiffness of spring, 50 pounds per square inch; length of stroke 30 inches; revolutions per minute, 84; piston diameter, 13½ inches.

84. The average breadth of an indicator diagram for one end of a piston is 1.58 inches, and for the other end it is 1.42 inches, and 1 inch represents 32 pounds per square inch. Piston, 12 inches diameter; crank, 1 foot long; revolutions per minute, 110. What is the indicated horse-power?

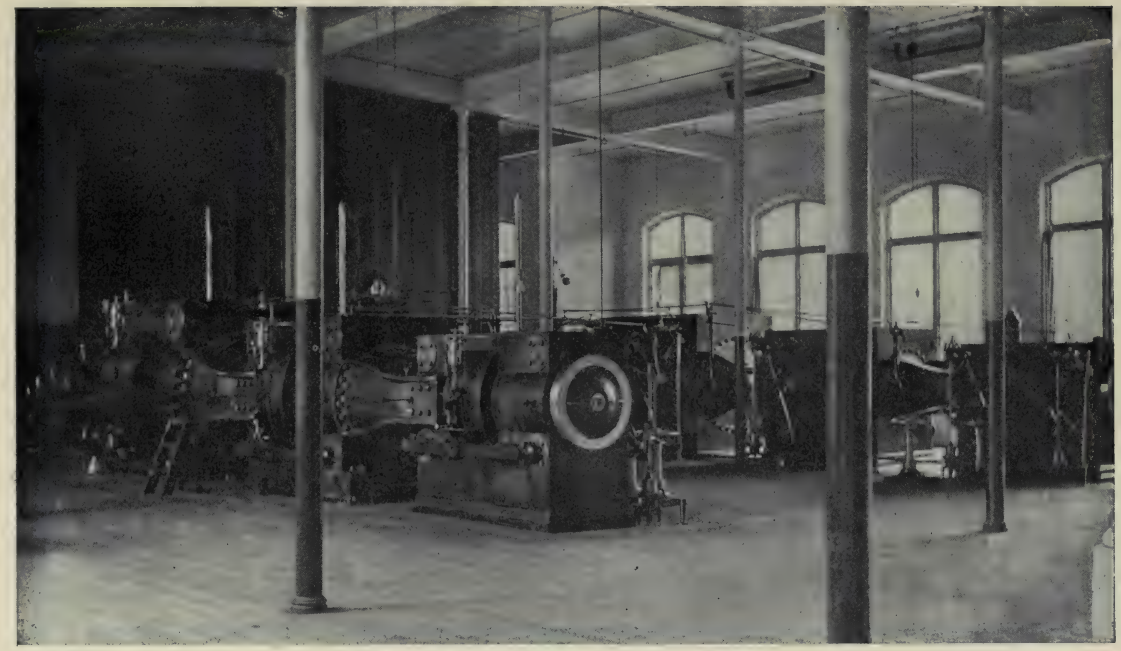
85. The cylinder of a steam-engine has an internal diameter of 3 feet; length of stroke, 6 feet; and it makes 10 strokes per minute. Under what effective pressure per square inch would it have to work in order that the piston may develop 125 horse-power?

The illustration of triple-expansion engines on opposite page accompanies Problem 86.

86. Four pairs of triple-expansion steam-engines are used to drive the cotton machinery of the largest Fall River corporation. One of these engines shown in illustration has cylinders $26\frac{1}{2}$ inches diameter, $36\frac{1}{2}$, and 54. The steam pressures are: In main pipe, 150 pounds per square inch; in receiver between high and intermediate cylinders, 40 pounds; in receiver between intermediate and low, 5 pounds. Vacuum is 27 inches. The mean effective pressures in the cylinders are respectively 54 pounds per square inch, $23\frac{1}{2}$ and $12\frac{1}{2}$. Length of stroke is 5 feet; piston speed, 660 feet per minute. Calculate the horse-power.

87. An engine is required to drive an overhead traveling crane for lifting a load of 30 tons at 4 feet per minute. The power is transmitted by means of $2\frac{1}{4}$-inch shafting, making 160 revolutions per minute. The length of the shafting is 250 feet; the power is transmitted from the shaft through two pairs of bevel gears (efficiency 90% each, including bearings), and one worm and wheel (efficiency 85%, including bearings). Taking the mechanical efficiency of the steam-engine at 80%, calculate the required horse-power of the engine.

88. An engine working at 50 horse-power is driven by steam at 75 pounds pressure acting on pistons in two cylinders. If the area of each piston is 72 square inches, and the length of stroke 2 feet, how many revolutions does the fly-wheel make per minute?



Triple-Expansion Engines in a Cotton Mill.

89. The steam-engine in use at the Worsted Weaving Mill of the Pacific Mills at Lawrence, Mass., is a Corliss type cross-compound with steam cylinders 19 and 36 inches diameter; stroke, 42 inches; revolutions, 100 per minute; mean effective pressures, 60 pounds and 13 pounds. Find how many looms weaving worsted dress-goods said engine will drive, each loom requiring $\frac{1}{4}$ horse-power.

90. A ship laden with coal must be unloaded at the rate of 22 tons of coal in 10 minutes. If the height of lift is 150 feet, what horse-power of engines will be required?

91. The fuel used in running a steam-engine is coal of such composition that the combustion of 1 pound produces heat sufficient to raise the temperature of 12 000 pounds of water 1° Fahr. It is found that $3\frac{1}{2}$ pounds of fuel are consumed per horse-power per hour. What is the efficiency of the entire apparatus?

92. A steam-engine uses coal of such composition that the combustion of 1 pound generates 10 000 British thermal units. If 40 pounds of coal are used per hour, and if the efficiency is 0.08, what horse-power is realized?

93. The cylinder of a Corliss-type steam-engine is 30 inches in diameter, stroke 48 inches, and it makes 85 revolutions per minute. The steam pressure being 90 pounds per square inch, what is the horse-power of the engine?

94. The piston of a steam-engine is 15 inches in diameter; its stroke is $2\frac{1}{2}$ feet, and it makes 20 revolutions per minute; the mean pressure of the steam on it is 15 pounds per square inch. How many foot-pounds of work are done by the steam per minute, and what is the horse-power of the engine?

95. An engine has a 6-foot stroke, the shaft makes 30 revolutions per minute, the average steam pressure is 25 pounds per square inch. Required the horse-power when the area of the piston is 1 800 square inches, the modulus of the engine being $\frac{11}{12}$.

96. The diameter of a steam-engine cylinder is 9 inches; the length of crank, 9 inches; the number of revolutions per minute, 110; the mean effective pressure of the steam 35 pounds per square inch. Find the indicated horse-power.

97. The 21 horse-power gas engine of problem 51 has a 12-inch piston and 8-inch crank. When making 150 revolutions per minute with an explosion every 2 revolutions what will be the mean effective pressure during a cycle?

98. The area of a cross-section of the Charles River at Riverside, Massachusetts, is 408 square feet. The velocity of current as found by rod floats and current meter, April 17 and 22, 1902, was 1.12 feet per second. What would be the theoretic horse-power of this quantity of water at the Waltham dam, which gives a fall of 12.58 feet?

Water-Power Canal and Mills, Manchester, N. H.

In water-power problems use

Work = force × distance

(area × velocity × $62\frac{1}{2}$)

Force = pounds of water flowing

Distance = height of dam or available fall

Fig. 8. — Water-Power.

99. Find the useful horse-power of a water-wheel, supposing the stream to be 100 feet wide and 5 feet deep, and to flow with a velocity of $\frac{1}{2}$ foot per second; the height of the fall is 24 feet, and the efficiency of the wheel 70 per cent.

100. A small stream has mean velocity of 35 feet per minute, fall of 13 feet and a mean section of 5 feet by 2. On this stream is erected a water-wheel whose modulus is 0.65. Find the horse-power of the wheel.

101. On page 32 is shown the canal at Manchester, N.H., as it passes the mills of the Amoskeag Manufacturing Company. Width is 51 feet, depth of water 8.9 feet, velocity 1.13 feet per second. What quantity of water is flowing? The height of fall for the turbines being 27.3 feet, what is the theoretic horse-power?

102. The reaction turbines of problem 101 have an efficiency of 80 per cent; the electric generators, 90 per cent. What kilowatts are available?

103. In winter, if 2 feet of ice forms on this canal, and the velocity drops to 0.75 feet per second, and the available fall becomes 25.0 feet, what will be the kilowatts available?

104. The mean section of the Merrimac Canal just before it enters the mills of the Merrimac Manufacturing Company at Lowell, Mass., is 48.2 feet by 10.6 feet; mean velocity on Nov. 23, 1901, was 2.37 feet per second; the water-wheels had a net fall of 35.67 feet, and gave an efficiency of about 77 per cent. Find the number of broad looms weaving cotton sheetings that may be driven $2\frac{1}{2}$ looms requiring one horse-power.

105. The estimated discharge of the nine turbines at Niagara Falls in 1898 was 430 cubic feet per second for each turbine. The average pressure head on the wheels was that due to a fall of about 136 feet. Compute the actual horse-power available from all turbines, allowing an efficiency of 82 per cent.

106. The average flow over Niagara Falls is 270 000 cubic feet per second. The height of fall is 161 feet. In round numbers what horse-power is developed?

107. Calculate the horse-power that can be obtained for one minute from an accumulator which makes

one stroke in a minute and has a ram of 20 inches diameter, 23 feet stroke, loaded to a pressure of 750 pounds per square inch.

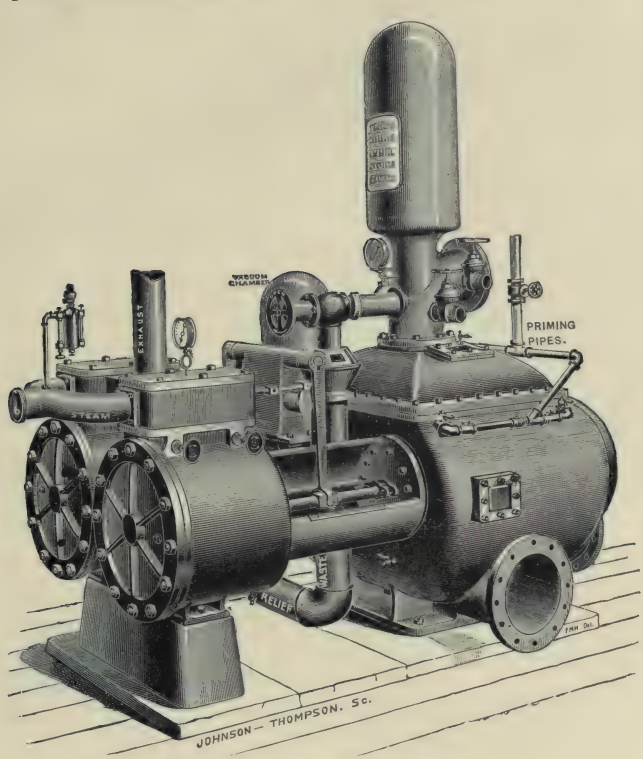


Fig. 9. An Underwriter Fire-Pump with Standard Fittings.

108. A fire-pump for protection of a 50 000-spindle cotton-mill will deliver 1 000 gallons of water per minute at 100 pounds pressure. Large boiler capa-

city is required for such a fire-pump and for the above size 150 horse-power would be used. What portion of this boiler capacity would be required in actual work of delivering water?

$$\text{Work of pumping} = \text{force} \times \text{distance}$$

$$\text{Force} = 1\,000 \times 8\tfrac{1}{3} \text{ pounds per minute}$$

$$\text{Distance} = 100 \times 2.304 \text{ feet head (1 pound} = 2.304 \text{ ft.)}$$

$$\therefore \text{Work of pumping} = 1\,000 \times 8\tfrac{1}{3} \times 100 \times 2.304$$

$$= 1\,920\,000 \text{ foot-pounds per minute}$$

$$= 58.3 \text{ horse-power}$$

$$\text{Portion of boiler used} = \frac{58.3}{150}$$

$$= 0\ 39, \text{ or about one-third}$$

109. An Underwriter fire-pump to protect an average-sized factory will deliver four streams of water through $1\frac{1}{8}$-inch smooth nozzles with pressure at base of play pipes of 50 pounds per square inch. This would correspond to a discharge of 1 060 gallons per minute. Loss of pressure through nozzle can be neglected; and loss in quantity of discharge by slippage, short strokage, and so on, will be about 10 per cent. Find the work done by the pump.

110. A pump of medium size used for fire protection of a factory will deliver three $1\frac{1}{8}$-inch fire streams, or 750 gallons per minute at 80 pounds pressure. A boiler should be provided large enough to allow 70 per cent of its capacity to remain as extra. What should be the nominal horse-power of boiler?

111. A Silsby steam fire-engine delivers water through a siamese nozzle that is 2 inches in diameter, with a pressure of 80 pounds per square inch and a mean velocity of 106 feet per second. Find (1) the number of cubic feet discharged per second; (2) the weight of water discharged per minute; (3) the work possessed by each pound of water due to 80 pounds pressure; (4) the horse-power of the engine required to drive the pump, assuming the efficiency to be 70 per cent.

112. At the Chestnut Hill High-Service Pumping Station (Boston) for the month of October, 1904, Engine No. 4 pumped 950 780 000 gallons of water; average lift was 130.63 feet; total time of pumping, 744 hours. What average horse-power was developed?

113. The amount of coal burned during the month was 783 148 pounds. How many foot-pounds of work were done for every 100 pounds of coal burned, that is, what was the Duty of the pumping-engine?

114. The ordinary fire-engine when in full operation burns soft coal, and will consume in an hour about 60 pounds per fire-stream of 250 gallons per minute. Therefore at the 70-million dollar fire in Baltimore, February, 1904, a 500-gallon engine that was running 30 hours, before the fire was under control, consumed how many pounds of coal?

115. Find the useful work done each second by a fire-engine which discharges water at the rate of 500

gallons per minute against a pressure of 100 pounds per square inch.

116. There were 6 000 cubic feet of water in a mine of 60-fathom depth when a 50-horse-power pump began to pump it out. It took 5 hours to empty it. Find the number of cubic feet of water that ran into the mine during the 5 hours, supposing one-fourth of the work of the pump to have been wasted.

117. Find the horse-power necessary to pump out the Saint Mary's Falls Canal Lock, Sault Ste. Marie, in 24 hours, the length of the lock being 500 feet, width 80 feet, and depth of water 18 feet, the water being delivered at a height of 42 feet above the bottom of the lock.

118. The mean section of the branch of the First Level Canal at the headgates of No. 1 Mill, Whiting Paper Co., Holyoke, Mass., is 78 feet wide by 14 deep; from this canal to the Second Level there is a fall of 20 feet, but about 2 feet is lost in penstock and tail-race; velocity of flow in canal during the daytime is 0.20 feet per second, and the turbines that are driven have an efficiency of 77%. Find how many 96-inch Fourdrinier Paper Machines can be driven, each machine requiring 100 horse-power.

119. What is the horse-power of a stream that passes through a section of 6 square feet at the rate of $2\frac{1}{2}$ miles an hour, and has a water-fall of 18 feet?

120. What horse-power is involved in lowering by 2 feet the level of the surface of a lake 2 square miles in area in 300 hours, the water being lifted to an average height of 5 feet?

121. Taking the average power of a man as $\frac{1}{10}$th of a horse-power, and the efficiency of the pump used as 0.4, in what time will 10 men empty a tank of 50 feet × 30 feet × 6 feet filled with water, the lift being an average height of 30 feet?

122. A shaft 560 feet deep and 5 feet in diameter is full of water. How many foot-pounds of work are required to empty it, and how long would it take an engine of $3\frac{1}{2}$ horse-power to do the work?

123. Required the number of horse-power to raise 2 200 cubic feet of water an hour from a mine whose depth is 63 fathoms.

124. What weight of coal will an engine of 4 horse-power raise in one hour from a pit whose depth is 200 feet?

125. A cut is being made on a 4-inch wrought-iron shaft revolving at 10 revolutions per minute; the traverse feed is 0.3 inch per revolution; the pressure on the tool is found to be 435 pounds. What is the horse-power expended at the tool? How much metal is removed per hour per horse-power when the depth of cut is .06 inch, the breadth .06 inch (triangular section)?

126. A man rides a bicycle up a hill whose slope is 1 in 20 at the rate of 4 miles an hour. The weight of man and machine is $187\frac{1}{2}$ pounds. What work per minute is he doing?

127. At the top of the hill the bicyclist referred to in example 126 is met by a strong head-wind, and he finds that he has to work twice as hard to keep the same rate of 4 miles an hour on the level. What force is the wind exerting against him?

128. A bicyclist works at the rate of one-tenth of a horse-power, and goes 12 miles an hour on the level. Prove that the constant resistance of the road is 3.125 pounds.

Prove that up an incline of 1 vertical to 50 horizontal the speed will be reduced to about 5.8 miles per hour, supposing that the man and machine together weigh 168 pounds.

129. A man rows a miles per hour uniformly. If R pounds be the resistance of the water, and P foot-pounds of useful work are done at each stroke, find the number of strokes made per minute.

130. The resistance offered by still water to the passage of a certain steamer at 10 knots an hour is 15 000 pounds. If 12% of the engine power is lost by "slip" — in pushing aside and backward the water acted on by the screw or paddle — and 8% is lost in friction of machinery, what must be the total horse-power of the engines?

131. The United States warship Columbia has a speed of 23 knots, with an indicated horse-power of 22 000. Find the resistance offered to her passage.

132. The rise and fall of the tide at Boston, Mass., is about 9 feet. If the in-coming water for one square mile of ocean surface could be stored and its potential energy used during the next 6 hours with an average fall of 3 feet, what horse-power would be available?

133. A nail 2 inches long was driven into a block by successive blows from a hammer weighing 5.01 pounds; after one blow it was found that the head of the nail projected 0.8 inches above the surface of the block; the hammer was then raised to a height of 1.5 feet and allowed to fall upon the head of the nail, which, after the blow, was found to be 0.46 inches above the surface. Find the force which the hammer exerted upon the nail at this blow.

134. A 500-volt motor drives a 10-ton car up a 5 per cent grade at a speed of 12 miles per hour: 75 per cent of the work of the motor is usefully expended. What electric current, expressed in amperes, will be required?

$$\underset{\text{of motor}}{\text{Work}} \times 0.75 = \underset{\text{of lifting car}}{\text{work}}$$

135. The speed of the "Exposition Flyer" on the Lake Shore and Michigan Southern Railroad, when running at its maximum, is 100 miles per hour. At

that speed what pull by the engine would represent one horse-power? What pull when running at 50 miles an hour?

136. An express train of weight 250 tons covers 40 miles in 40 minutes. Taking the train resistances on a level track to be 20 pounds per ton at this speed find the horse-power that engine must develop.

137. A train goes down a grade of 1% for a distance of 1 mile, steam throttle being kept shut; it then runs up an equal grade with its acquired velocity for a distance of 500 yards before stopping. Find the total resistances, frictional or other, in pounds per ton, which are stopping it.

138. A caboose and three cars break away from a freight train and "coast" down a grade of 2 in 100 for a distance of 1 mile; then brakes are applied and the cars stopped in 200 feet. Frictional resistances over whole distance being 15 pounds per ton, what are the brake resistances per ton?

$$\text{Work}_{\text{down grade}} = \text{Work}_{\text{of friction}} + \text{Work}_{\text{of brakes}}$$

139. The Baltimore and Ohio Railroad has now in its service (1906) six electric locomotives. Two of recent construction are used in handling eastbound freight trains with steam locomotives through the city of Baltimore, which includes a distance of about two miles of tunnel. These locomotives can start and accelerate on a level track a train of 3 000 tons weight with a current consumption of 2 200 amperes, which

is supplied from a power station at 560 volts, but reaches the locomotives through booster stations and a storage battery at 625 volts. What horse-power do they thus develop?

140. These electric locomotives will draw on a 1% grade a freight train of 1 400 tons weight at 10 miles an hour. Frictional resistances being 20 pounds per ton what amperes are necessary with voltage of 625?

ENERGY.

141. A train of 150 tons is running at 50 miles an hour. What brake force is required to stop it in a quarter of a mile on a down grade of 2%, frictional resistance being 15 pounds per ton?

Work possessed by train + Work gained in $\frac{1}{4}$ mile = Work of brakes + Work of resistances

Work possessed by train = force × distance

Force = 150 tons

Distance = ?

The distance is found by determining the vertical height that a body would have to fall in order to acquire a velocity of 50 miles an hour. 30 miles an hour = 44 feet per second. Therefore the velocity is $\frac{50}{30} \times 44 = \frac{220}{3}$ feet per second. Now to acquire a velocity of $\frac{220}{3}$ feet per second a body would fall a vertical distance that can be found from a fundamental formula of falling bodies,

$v = \sqrt{2gh}$, in which g* varies for different localities

$\frac{220}{3} = 8\sqrt{h}$

$h = 84.2$ feet.

That is, if the train had fallen by the action of gravity from a vertical height of 84.2 feet it would have a velocity of $\frac{220}{3}$ feet per second, or 50 miles an hour. Work can now be analyzed as in previous problems.

Work possessed by train = 150×84.2 foot-tons

Work gained in $\frac{1}{4}$ mile = $150 \times (\frac{2}{100} \times \frac{5280}{4})$

* The value of g for London is 32.19 feet per second per second; for San Francisco, 32.15; for Chicago, 32.16; for Boston, 32.16. Practical limiting values for the United States are 32.186 at sea level for latitude 49°; and 32.089, latitude 25° and 10 000 feet above sea level. In this book the value 32 is used for g so that computations may be shortened. In many cases the table on page 190 will be of assistance. The values of $\sqrt{2gh}$ there given are based on g as 32.16.

142. In the Westinghouse brake tests (Jan., 1887), at Weehawken, a passenger-train moving 22 miles an hour on a down grade of 1% was stopped in 91 feet. There was 94% of the train braked. Taking the frictional resistance as 8 pounds per ton, find the net brake resistance per ton on the part of the train that was braked, and the grade to which this is equivalent.

143. A freight-car weighing 20 000 pounds requires a net pull of 10 pounds per ton to overcome frictional resistance. If "kicked" to a level side track with velocity of 10 miles per hour, how far will it run before stopping?

144. A cake of ice weighing 150 pounds slides down a chute the height of which is 25 feet; it reaches the foot of the shute with a velocity of 30 feet per second. During the motion how many foot-pounds of energy must have been lost?

145. A ship and its cradle that weigh 5 000 tons slides down ways that slope 1 foot in 20 to the horizontal; frictional resistances amount to a constant retarding force of 100 tons. What would be the equivalent height of fall that would produce the same velocity as the ship possesses when she takes the water 150 feet down the ways?

146. If the resistance of the water, anchors, and stop ropes amount to a constant force of 50 tons, how far will the ship of the preceding problem run after she takes the water?

147. A six-inch rapid-fire gun discharges 5 projectiles per minute, each of weight 100 pounds, with a velocity of 2 800 feet per second. What is the horse-power expended?

Consider from what vertical height a body would fall to have a velocity of 2 800 feet per second. ($v = \sqrt{2gh}$).

148. A railway car of 4 tons, moving at the rate of 5 miles an hour, strikes a pair of buffers which yield to the extent of 6 inches. Find the average force exerted upon them.

$$\underset{\text{possessed by train}}{\text{Work}} = \underset{\text{by buffers}}{\text{work}}$$

149. What is the kinetic energy of a $2\frac{1}{2}$-ton cable car moving at 6 miles per hour, loaded with 36 passengers, each of average weight 154 pounds? If stopped in 2 seconds, what is the average force?

150. What is the kinetic energy of an electric car weighing $2\frac{1}{2}$ tons, moving at 10 miles an hour, and loaded with 50 passengers, of average weight 150 pounds?

151. The weight of a ram is 600 pounds, and at the end of a blow it has a velocity of 40 feet per second. What work is done in raising it?

152. Find the horse-power of a man who strikes 25 blows per minute on an anvil with a hammer of weight 14 pounds, the velocity of the hammer on striking being 32 feet per second.

153. A blacksmith's helper using a 16-pound sledge strikes 20 times a minute and with a velocity of 30 feet per second. Find his rate of work.

154. A ball weighing 5 ounces, and moving at 1 000 feet per second, pierces a shield, and moves on with a velocity of 400 feet per second. What energy is lost in piercing the shield?

155. A shot of 1 000 pounds moving at the rate of 1 600 feet per second strikes a fixed target. How far will the shot penetrate the target, exerting upon it an average pressure equal to a weight of 12 000 tons?

156. A bullet weighing 1 ounce leaves the mouth of a rifle with a velocity of 1 500 feet per second. If the barrel be 4 feet long, calculate the mean pressure of the powder, neglecting all friction.

157. The bullet referred to in the preceding problem penetrates a sand bank to the depth of 3 feet. What is the mean pressure exerted by the sand?

158. An 8-hundred weight shot leaves a 40-ton gun with velocity of 2 000 feet per second: the length of the gun is 20 feet. What is the average force of the powder?

159. A 2-ounce bullet leaves the barrel of a gun with a velocity of 1 000 feet per second. Find the work stored up in the bullet, and the height from which it must fall to acquire that velocity.

160. What is the kinetic energy of a 5-hundred weight projectile fired with a velocity of 2 000 feet per second?

161. An 8-inch projectile, weight 250 pounds, strikes a sand butt going 2 000 feet per second, and

is stopped in 25 feet. If the resistance is uniform, what is its value in pounds?

162. A hammer weighing 1 pound has a velocity of 20 feet per second at the instant it strikes the head of a nail. Find the force which the hammer exerts on the nail if it is driven into the wood $\frac{1}{16}$ of an inch.

163. A fly-wheel weighs 10 000 pounds, and is of such a size that its mass may be treated as if concentrated on the circumference of a circle 12 feet in radius. What is its kinetic energy when moving at the rate of 15 revolutions a minute?

164. How many turns would the above fly-wheel make before coming to rest, if the steam were cut off, and it moved against a friction of 400 pounds exerted on the circumference of an axle 1 foot in diameter?

165. A fly-wheel on a 21-horse-power gas engine of nominal speed 150 revolutions per minute, must store what energy to provide for an increase or decrease in speed of 3 revolutions per minute?

166. The fly-wheel of a 4-horse-power engine running at 75 revolutions per minute is equivalent to a heavy rim of mean diameter 2 feet 9 inches, and weight 500 pounds. What is the ratio of the work stored in the fly-wheel to the work developed in a revolution?

167. A 3 horse-power stamping machine presses down once in every 2 seconds; its speed fluctuates from 80 to 120 revolutions per minute; and to provide for this fluctuation the fly wheel stores $\frac{7}{8}$ths of

all the energy supply for 2 seconds. What energy is thus stored per revolution?

168. A nozzle discharges a stream 1 inch in diameter with a velocity of 80 feet per second. (*a*) How much work is possessed by the water that flows out each minute? (*b*) If this energy could all be utilized by a water-wheel, what would be its power?

169. Suppose that the above nozzle drives a water-wheel connected with a pump which lifts water 20 feet. If the efficiency of the whole apparatus is 0.48, how much water would be lifted per minute?

170. An impulse water-wheel must provide $3\frac{1}{2}$ useful horse-power; efficiency of wheel is 85%; water-pressure is 60 pounds per square inch; what size nozzle should be used — to the nearest eighth of an inch?

$$\begin{aligned} \text{Work} &= \text{force} \times \text{distance} \\ \text{Force} &= \text{area} \times \text{velocity} \times 62\tfrac{1}{2} \\ &= \text{area} \times 8\sqrt{60 \times 2.304} \times 62\tfrac{1}{2} \\ \text{Distance} &= 60 \times 2.304 \text{ feet.} \end{aligned}$$

171. The fire streams shown on the next page are being delivered through 100 feet of cotton rubber-lined hose with nozzles $1\frac{1}{8}$ inches in diameter. The full pressure at end of nozzle is 50 pounds per square inch. What horse-power is the fire-pump thus delivering?

172. Through the 100 foot lines of hose there is a large loss of pressure. At hydrant the full pressure is 75 pounds; at the nozzle 50 pounds. What horse-power is thus lost?

Force illustrated by two fire streams being delivered by the pump service of the large cotton mills of B. B. & R. Knight at Natick, Rhode Island. One stream is being held by men in correct position, the other by men who have been crowded into an awkward and dangerous position. Pressure shown on the gauge at the hydrant was 75 pounds per square inch.

II. FORCES.

FORCES ACTING AT A POINT.

173. An acrobat weighing 150 pounds stands in the middle of a tight rope 40 feet long and depresses it 5 feet. Find the tension in the rope caused by his weight.

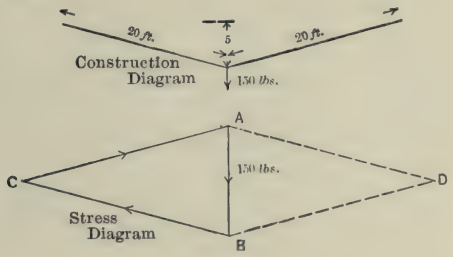


Fig. 11.

Draw the construction diagram showing the rope, the known force of 150 pounds and an arrow to indicate its direction. Then draw the stress diagram. Lay off 150 pounds parallel to the known force and at a scale of 1 inch = 60 pounds; from one end of this line AB draw a full line parallel with one of the forces; from the other end a line parallel with the other force. Complete the parallelogram by drawing free hand the dotted parallel lines AD and BD. Put arrows on forces beginning with the known force of 150 pounds — this positively acts downward — then the other arrows will follow in order around the full-line triangle B to C and C to A. Thus AB has become a diagonal of the parallelogram and is the balancing force, or what is more properly known as the equilibriant. If this

force was acting in the opposite direction it would be the resultant of the other two forces.

The full-line triangle ABC constitutes the triangle of forces. It is the keynote to the solution of many problems in Forces. The magnitudes and directions of the forces can be found by scaling from this triangle, or by computations involving similar triangles, thus, geometry or trigonometry.

Forces-At-A-Point problems therefore can be solved as follows:

Draw a construction diagram showing dimensions and loads.
Draw a stress diagram.
First, the known force.
Complete the parallelogram.
Put arrows on full-line triangle.
Scale or compute the stresses.

174. A speed-buoy is thrown into the water behind a ship, and the pull on the buoy by the water is 60 pounds. The two ropes that connect the buoy with the ship make an angle of 15° at their point of attachment. Find the stresses on the ropes.

175. Two men pull a body horizontally by means of ropes. One exerts a force of 28 pounds directly north, the other a force of 42 pounds in direction N. 42°E. What single force would be equivalent to the two?

176. Three cords are knotted together; one of these is pulled to the north with a force of 6 pounds, another to the east with a force of 8 pounds. With what force must the third be pulled to keep the whole at rest?

177. Two persons lifting a body exert forces of 44 pounds and 60 pounds on opposite sides of the ver-

tical, but each with an inclination of 28°. What single force would produce the same effect?

178. A force of 50 units acts along a line inclined at an angle of 30° to the horizon. Find, by construction or otherwise, its horizontal and vertical components.

179. Explain the boatman's meaning when he says that greater force is developed when a mule hauls a canal boat with a long rope than with a short one. Is the same true of a steam-tug when towing a four-master?

180. Two strings, one of which is horizontal, and the other inclined to the vertical at an angle of 30°, support a weight of 10 pounds. Find the tension in each string.

181. Two forces of 20 pounds, and one of 21 act at a point. The angle between the first and second is 120°, and between the second and third, 30°. Find the resultant.

182. Forces of 9 pounds, 12, 13, and 26, act at a point so that the angles between the successive forces are equal. Find their resultant.

183. A weightless rod, 3 feet long, is supported horizontally, one end being hinged to a vertical wall, and the other attached by a string to a point 4 feet above the hinge; a weight of 120 pounds is hung from the end supported by the string. Calculate the tension in the string, and the pressure along the rod.

184. A weight of 100 pounds is fixed to the top of a weightless rod or strut 5 feet long whose lower end rests in a corner between a floor and a vertical wall, while its upper end is attached to the wall by a horizontal wire 4 feet long. Calculate the tension in the wire, and the thrust in the rod.

185. A rod AB is hinged at A and supported in a horizontal position by a string BC making an angle of 45° with the rod; the rod has a weight of 10 pounds suspended from B. Find the tension in the string and the force at the hinge. (The weight of the rod can be neglected.)

186. A simple triangular truss of 30 feet span and 5 feet depth supports a load of 4 tons at the apex. Find the forces acting on rafters and tie rod.

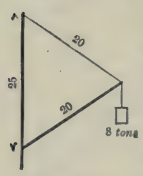


Fig. 12.

187. A derrick is set as shown in sketch, the load being 8 tons. Find the stress in the boom and the tackle.

188. A stiff-leg steel derrick, with mast 55 feet high, boom 85 feet long, set with tackle 40 feet long, as shown in cut, is raising two boilers of 50 tons weight. Find stresses in boom and tackle. (See illustration on page 55.)

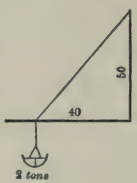


Fig. 13.

189. Find the stress in tackle and compression in boom of towers for six-master shown on page 24 when bucket, weighing with its load 2 tons, is set in position shown by Fig. 13.

Fig. 14.

190. A balloon capable of raising a weight of 360 pounds is held to the ground by a rope which makes an angle of 60° with the horizon. Determine the tension of the rope and the horizontal pressure of the wind on the balloon.

191. A uniform beam 10 feet long, weighing 80 pounds, is suspended from a horizontal ceiling by two strings attached at its ends, and at points 16 feet apart in the ceiling. Find the tension in each string.

192. A boat is towed along a canal 50 feet wide, by mules on both banks; the length of each rope from its point of attachment to the bank is 72 feet:

the boat moves straight down the middle of the canal. Find the total effective pull in that direction, when the pull on each rope is 800 pounds.

193. A boat is being towed by a rope making an angle of 30° with the boat's length; the resultant pressure of the water and rudder is inclined at 60° to the length of the boat, and the tension in the rope is equal to the weight of half a ton. Find the resultant force in the direction of the boat's length.

194. In a direct-acting steam-engine the piston-pressure is 22 500 pounds; the connecting-rod makes a maximum angle of 15° with the line of action of the piston. Find the pressure on the guides.

195. A man weighing 160 pounds sits in a loop at the end of a rope 10 feet 3 inches long, the other end being fastened to a point above. What horizontal force will pull him 2 feet 3 inches from the vertical, and what will then be the pull on the rope?

196. A man weighing 160 pounds sits in a hammock suspended by ropes which are inclined at 30° and 45° to vertical posts. Find the pull in each rope.

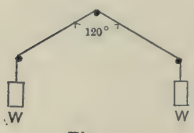


Fig. 15.

197. Two equal weights, W, are attached to the extremities of a flexible string which passes over three tacks arranged in the form of an isosceles triangle with the base horizontal, the vertical angle at the upper tack being 120°. Find the pressure on each tack.

198. A rod AB 5 feet long, without weight, is hung from a point C by two strings, which are attached to its ends and to the point; the string AC is 3 feet long, and the string BC 2 feet; a weight of 2 pounds is hung from A and a weight of 3 pounds from B. Find the tension of the strings and the condition that these may be in equilibrium.

199. A weight of 10 pounds is suspended by two strings, 7 and 24 inches long, the other ends of which are fastened to the extremities of a rod 25 inches in length. Find the tension of the strings when the weight hangs immediately below the middle point of the rod.

200. AB is a wall, and C a fixed point at a given perpendicular distance from it; a uniform rod of given length is placed on C with one end against AB. If all the surfaces are smooth, find the position in which the rod is in equilibrium.

201. AB is a uniform beam weighing 300 pounds. The end A rests against a smooth vertical wall, the end B is attached to a rope CB. Point C is vertically above A, length of beam is 4 feet, rope 7 feet. Represent the forces acting, and find the pressure against the wall and the tension in the rope.

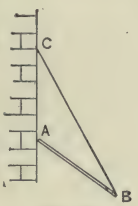


Fig. 16.

202. A wagon weighing 2 200 pounds rests on a slope of inclination 30°. What are the equivalent forces parallel and perpendicular to the plane?

203. AB is a rod that can turn freely round one end A; the other end B rests against a smooth inclined plane. In what direction does the plane react upon the rod? Illustrate your answer by a diagram showing the rod, the plane, and the reaction.

204. A wagon weighing 2 tons is to be drawn up a smooth road which rises 4 feet vertically in a distance of 32 feet horizontally by a rope parallel to the road. What must the pull of the rope exceed in order that it may move the wagon?

205. What weight can be drawn up a smooth plane rising 1 in 5 by a pull of 200 pounds (*a*) when the pull is parallel with the plane? (*b*) when it is horizontal?

206. A horse is attached to a dump-car by a chain, which is inclined at an angle of 45° to the rails; the force exerted by the horse is 672 pounds. What is the effective force along the rails?

207. The angle of inclination of a smooth inclined plane is 45°: a force of 3 pounds acts horizontally, and a force of 4 pounds acts parallel to the plane. Find the weight which they will be just able to support.

208. A body rests on a plane of height 3 feet, length 5 feet. If the body weighs 14 pounds, what force acting along the plane could support it, and what would be the pressure on the plane?

209. A number of loaded trucks each containing one ton, standing on a given part of a smooth tramway, where the inclination is 30°, support an equal number of empty trucks on another part, where the inclination is 45°. Find the weight of a truck.

210. Two planks of lengths 7 yards and 6 yards rest with one end of each on a horizontal plane, the other ends in contact above that plane; two weights are supported one on each plank, and are connected by a string passing over a pulley at the junction of the planks; the weight on the first plank is 21 pounds. What is the weight on the other, friction not being considered?

211. The weight of a wheel with its load is 2 tons, diameter of wheel 5 feet. Find the least horizontal force necessary to pull it over a stone 4 inches high. (When the wheel begins to rise three forces are acting: P, W, and R the reaction. It is required to find P.)

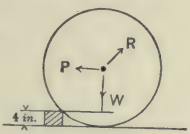


Fig. 17.

212. A rectangular box, containing a 200-pound ball, stands on a horizontal table, and is tilted about one of its lower edges through an angle of 30.° Find the pressure between the ball and the box.

213. An iron sphere weighing 50 pounds is resting against a smooth vertical wall and a smooth plane which is inclined 60° to the horizon. Find the pressure on the wall and plane.

214. A beam weighing 400 pounds rests with its ends on two inclined planes whose angles of inclination to the horizontal are 20° and 30°. Find the pressures on the planes.

215. A thread 14 feet long is fastened to two points A and B which are in the same horizontal line and 10 feet apart; a weight of 25 pounds is tied to the thread at a point P so chosen that AP is 6 feet — therefore BP is 8 feet long. The weight being thus suspended, find by means of construction or otherwise, what are the tensions of the parts AP and BP of the thread.

216. AC and BC are two threads 4 feet and 5 feet long, respectively, fastened to fixed points A and B, which are in the same horizontal line 6 feet apart; a weight of 50 pounds is fastened to C. Find, by means of a line construction drawn to scale, the pull it causes at the points A and B. Each of the threads AC and BC is, of course, in a state of tension. What are the forces producing the tension?

217. A boiler weighing 3 000 pounds is supported by tackles from the fore and main yards. If the tackles make angles of 25° and 35° respectively with the vertical, what is the tension of each?

218. A piece of wire 26 inches long, and strong enough to support directly a load of 100 pounds, is attached to two points 24 inches apart in the same horizontal line. Find the maximum load that can be

suspended at the middle of the piece of wire without breaking it.

219. A picture of 50 pounds weight hanging vertically against a smooth wall is supported by a string passing over a smooth hook; the ends of the string are fastened to two points in the upper rim of the frame, which are equidistant from the center of the rim, and the angle at the peg is 60°. Find the tension in the string.

220. A weight W attached by two connecting cords of lengths a and b to two fixed points A and B, and separated by a horizontal interval c, are in equilibrium under the action of gravity. Required the stresses P and Q in the cords.

221. Two equal rods AB and BC are loosely jointed together at B. C and A rest on two fixed supports in the same horizontal line, and are connected by a cord equal in length to AB. If a weight of 12 pounds be suspended from B, what is the pressure produced along AB and BC, and the tension in the cord?

222. Two spars are lashed together so as to form a pair of shears as shown in sketch. They stand with their "heels" 20 feet apart, and would be 40 feet high when vertical. What is the tension in the guy and thrust in the legs when a load of 30 tons is being lifted?

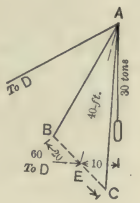


Fig. 18.

Suppose that a single leg should replace the

two spars. The stress can easily be found in this imaginary leg by considering that at A, in this plane, three forces meet, — the imaginary leg, the back guy, and the vertical load of 30 tons. Then consider the three forces at A in the plane of the legs, and thus find the stresses in the two equal spars.

223. When the spars become vertical what stresses will exist for the load of 30 tons?

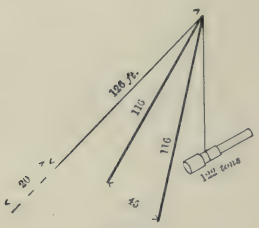


Fig. 19.

224. Figs. 19–20 show a pair of shears erected at Sparrow's Point, Md., for the Maryland Steel Company. The two front legs are hollow steel tubes 116 feet long, and inclined 35 feet out of the vertical. The back leg is 126 feet long, and is connected to hydraulic machines for operating the shears. How much are the forces acting in these legs when a Krupp gun weighing 122 tons is being lifted?

225. Each leg of a pair of shears is 50 feet long. They are spread 20 feet at the foot. The back stay is 75 feet long. Find the forces acting on each member when lifting a load of 20 tons at a distance of 20 feet from the foot of the shear legs, neglecting the weight of structure.

226. Shear legs each 50 feet long, 30 feet apart on horizontal ground, meet at point C, which is 45 feet vertically above the ground; stay from C is inclined

Fig. 20.

at 40° to the horizon ; a load of 10 tons hangs from C. Find the force in each leg and stay.

227. A vertical crane post is 10 feet high, jib 30 feet long, stay 24 feet long, meeting at a point C. There are two back stays making angles of 45° with the horizontal ; they are in planes due north and due

Dipper Dredge "Pan American."

west from the post. A weight of 5 tons hangs from C. Find the forces in the jib and stays — 1st, when C is southeast of the post; 2d, when C is due east; 3d, when C is due south.

228. The view on opposite page shows one of the largest dipper dredges ever built, the "Pan American," constructed at Buffalo in 1899 for use on the Great Lakes. An A-frame, the legs of which are 57 feet long and 40 feet apart at the bottom, is held at the apex by four cables which are 100 feet long. The boom is 53 feet long and weighs 30 tons. The handle, which weighs about 4 tons, is 60 feet long, and carries on its end a dipper weighing 16 tons, which will dredge up $8\frac{1}{4}$ cubic yards, or about 12 tons, of material at one load.

The dipper is operated by a wire rope passing over a pulley on the outward end of the boom. In the position represented by the outline sketch, the boom is inclined to the water surface at an angle of 30°, the dipper is carrying the full load, and the handle is in a horizontal position with its middle point supported at a point on the boom 23 feet from the foot of the boom. The apex of the A-frame is vertically above the foot of the boom. Compute the forces acting in the 100-foot

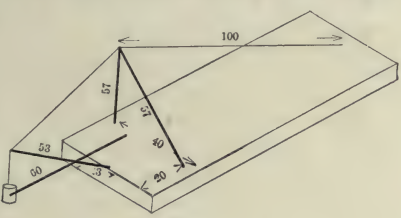


Fig. 21.

back-stays (considering them to be one rope, in position as per sketch), in the legs of the A-frame, in the boom, and in the wire rope which raises the dipper.

229. A tripod whose vertex is A, and whose legs are AB, AC, AD, of lengths 8 feet, 8.5, and 9 respectively, sustains a load of 2 tons. The ends B, C, D, form a triangle whose sides are BC 7 feet, CD 6 feet, BD 8 feet. Find the stress in each leg.

Sketch the figure and put on the dimensions. Then draw to scale the base BCD, and in this horizontal plane locate the vertices A′, A″, and A‴ of the three faces of the pyramidal-shaped figure that is formed by the legs of the tripod. Perpendiculars drawn from A′, A″ and A‴ to their respective sides of the triangle BCD will locate at their intersection the projection of vertex A. Now pass a vertical plane, for example, through AB and the load of 2 tons; note the intersection E with line CD. AE can be considered as an imaginary leg, and the stress in it can be graphically determined as heretofore, also the stress in AC and AD.

30. A tripod with 8-foot legs is to be used for lowering a 2-ton water-pipe. How far apart can the bottoms of legs be spread, if in an equilateral triangle, so that not over 1 ton stress will come on each leg?

231. A chandelier of weight 500 pounds is to hang under the middle of a triangle 12 feet $\times$ 8 $\times$ 8. Two of the chains are to be 20 feet long. What should be the length of the third chain? What stresses would exist in chains?

232. ABCD is a square; forces of 1 pound, 6, and 9 act in directions AB, AC, and AD respectively. Find the magnitude of their resultant.

233. A, B, C, D, are the angular points of a square taken in order; three forces act on a particle at A, viz. one of 7 units from A to B, a second of 10 units from D to A, and a third of $5\sqrt{2}$ units along the diagonal from A to C. Find, by construction or otherwise, the resultant of these three forces.

234. Forces P, 2P, 3P, and 4P act along the sides of a square A, B, C, D, taken in order. Find the magnitude, direction, and line of action of the resultant.

235. A sinker is attached to a fishing-line which is then thrown into running water. Show by means of a diagram the forces which act on the sinker so as to maintain equilibrium.

236. A uniform rod 6 feet long, weighing 10 pounds, is supported by a smooth pin and by a string 6 feet long which is attached to the rod 1 foot from one end and to a nail vertically above the pin, 4 feet distant. Show by construction the position in which the rod will come to rest.

237. A light rod AB can turn freely round a hinge at A; it rests in an inclined position against a smooth peg near the end B; a weight is hung from the middle of the rod. Show in a diagram the forces which keep the rod at rest, and name them.

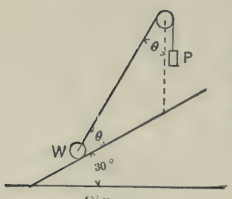


Fig. 22.

238. A weight W on a plane inclined 30° to the horizontal is supported as shown in cut. The angles θ being equal. Find the ratio of the power to the weight.

239. Discuss the action of the wind in propelling a sailing-vessel.

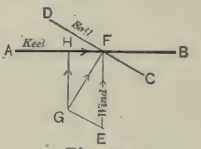


Fig. 23.

Let AB be the keel, CD the sail. Let the force of the wind be represented in magnitude and direction by EF. The component GF of EF, perpendicular to the sail, is the effective component in propelling the ship; the other component EG, parallel to the sail, is useless; but GF drives the ship forward and sidewise. The component GH of GF, perpendicular to AB, produces side motion, or leeway; and the other component HF, along the keel, produces forward motion, or headway.

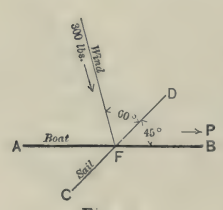


Fig. 24.

240. A sailing-boat is being driven forward by a force of 300 pounds as shown in Fig. 24. What force is P acting in direction of motion of the boat?

241. Discuss the action of the rudder of a vessel in counteracting leeway. Show that one effect of the action of the rudder is to diminish the vessel's motion.

242. A thread of length l has its ends fastened to two points in a line of length c, and inclined to the vertical with angle θ; a weight W hangs on the thread by means of a smooth hook. Find the position in

which the weight comes to rest and the tension in the thread.

243. A smooth ring weighing 40 pounds slides along a cord that is attached to two fixed points in a horizontal line. The distance between the points being one-half length of cord, find position in which weight will come to rest and the tension in the string near the points of attachment.

244. A small heavy ring A, which can slide upon a smooth vertical hoop, is kept in a given position by a string AB, B being the highest point of the hoop. Show that the pressure between the ring and the hoop is equal to the weight of the ring.

245. Draw a figure showing the mechanical conditions of equilibrium when a uniform beam rests with one extremity against a smooth vertical wall, and the other inside a smooth hemispherical bowl.

246. A ball 8 inches in diameter, weighing 100 pounds, rests on a plane inclined 30° to the horizon, and is held in equilibrium by a string 4 inches long attached to a sphere and to an inclined plane. Represent the forces acting, and find their values.

247. A uniform sphere rests on a smooth inclined plane, and is held by a horizontal string. To what point on the surface of the sphere must the string be attached? Draw a figure showing the forces in action,

248. A uniform bar of weight 20 pounds, length 12 feet, rests with one end inside a smooth hemispherical bowl, and is supported by the edge of the bowl with 2 feet of the bar outside of it. Draw the forces producing equilibrium, and find their values.

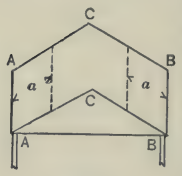


Fig. 25.

The stresses in a roof or bridge truss that carries a uniform load are best determined by finding in place of the uniform loads equivalent apex loads. And a fact that is often obscure to students is, that a part of this uniform load is not included in our computation of stresses. In the truss of Fig. 25 the portions *a* of uniform load are not included in the compressive stresses of A C and C B. This fact will be further understood by solving the problems that follow.

249. Two floor beams of 16 feet length meet at a post, Fig. 26. The load, 10 feet width of bay for each beam, is 150 pounds per square foot. What will be the load carried by the post? If it is found that the post must be removed so as to give better floor space the plan of Fig. 27 could be used. What would then be the stress in the short post (3 feet long), and in the two rods, and in the floor beams?

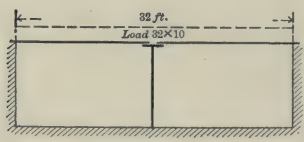


Fig. 26.

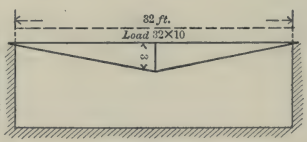
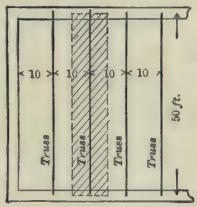


Fig. 27.

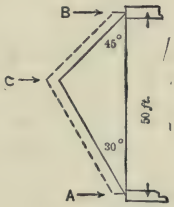
250. Now if the same conditions exist as in the preceding problem, except that the rods, instead of

being fastened to the ends of the beam are fastened to straps on the outside of the wall, what will then be the stresses in post, rods, and floor beam?

251. The slopes of a simple triangular roof-truss are 30° and 45°, and the span is 50 feet. The trusses are set 10 feet apart, and the weight of the roof covering and snow is 50 pounds per square foot of roof. Find the stresses in tie-rod and rafters.



Plan
Fig. 28.



Elevation
Fig. 29.

The load on any truss would be represented by the shaded area in Fig. 28. Find this load and then the apex loads A, C, and B, and observe that, according to explanation of preceding problems, the loads A and B do not enter into our computations. C alone is required. Having found C, the stresses in rafters can be determined. Then find the stress that each rafter transmits to the tie-rod.

252. In a roof of 32 feet span and height 12 feet the trusses are 10 feet apart, and the members EF, GH, come to the middle points of the rafters. If the weight of the roof-covering and snow is 60 pounds per square foot, find the apex loads AO, AB, and BC.

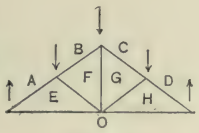


Fig. 30.

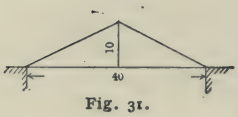


Fig. 31.

253. Find the stresses in the king-post truss of Fig. 31. Distance between trusses is 12 feet. There is a uniform load of 100 pounds per square foot of roof surface and 1 000 pounds at the foot of the post.

254. A king-post truss has a span of 18 feet and a rise of 9 feet. Compute the stresses due to a load of 14 000 pounds at the middle.

255. A floor beam 16 feet long and carrying a uniform load of 200 pounds per linear foot is trussed by rods that are $1\frac{1}{2}$ feet below middle of beam. Consider a joint at the middle and find stress in rod.

MOMENTS

The principles of Work can be used to solve nearly all problems that belong to the subject of Mechanics, but in certain classes of problems shorter methods are possible. In the following problems the principles of Moments can be used to advantage.

DEFINITION. — The Moment of a force about a point or axis is the product of the force times the perpendicular distance from the point to the line of action of the force; or, briefly, Moment is force × perpendicular.

Clockwise motion will be taken positive; the opposite direction, negative.

In beginning the solution of problems always state which point or axis the moments are taken about; thus, "Take moments about B," or "Moments about axis B."

256. A piece of shafting 10 feet long, and weighing 100 pounds, rests horizontally on two horses placed at its ends. A pulley weighing 75 pounds is keyed $2\frac{1}{3}$ feet from one end. How many pounds will a man have to lift at the other end to just raise it?

Fig. 32.

100 pounds, the weight of shaft, acts downward at the middle point; 75 pounds, the weight of pulley, acts downward at D, $2\frac{1}{3}$ feet from B. Find the required force acting upward.

Take moments about B,

$$+ P \times 10 - 100 \times 5 - 75 \times 2\tfrac{1}{3} = 0.$$

$$\therefore P = 67.5 \text{ pounds.}$$

257. A uniform lever is 18 inches long, and each inch in length weighs 1 ounce. Find the place of the fulcrum when a weight of 27 ounces at one end of the lever balances a weight of 9 ounces at the other end.

258. A lever 16 feet long balances about a point 4 feet from one end; if a weight of 120 pounds be attached to the other end, it balances about a point 6 feet from that end. Find the weight of the lever.

259. A light rod of length 3 yards has weights of 15 pounds and 3 pounds suspended at the middle and end respectively; it balances on a fulcrum. Find the position of the fulcrum, and the pressure on it.

260. A stiff pole 12 feet long sticks out horizontally from a vertical wall. It would break if a weight of 28 pounds were hung at the end. How far out along the pole may a boy of weight 112 pounds venture with safety?

261. A man pulls 100 pounds on the end of a 7-foot oar that has $2\frac{1}{2}$ feet inside the rowlock. What is the pressure on the rowlock, and resultant pressure causing the boat to move?

262. Find the propelling force on an eight-oared shell, if each man pulls his oar with a force of 56 pounds, and the length of the oar outside the rowlock is three times the length inside.

263. A light bar, 5 feet long, has weights of 9 pounds and 5 pounds suspended from its ends, and 10 pounds from its middle point. Where will it balance?

264. A weightless lever AB of the first order, 8 feet long, with its fulcrum 2 feet from B, has a weight of 5 pounds hung from A, and one of 17 pounds from B. From what point must a weight of 2.5 pounds be hung to keep the lever horizontal?

265. A weight of 100 pounds is supported by a rope which passes over a fixed pulley and is attached to a 12-foot lever at a point 2 feet from the fulcrum which is at the end. What weight must be suspended at the other end to keep the lever horizontal?

266. Eight sailors raise an anchor, of weight 2 688 pounds, by pulling on the spokes of a capstain which has a radius of 14 inches. If they all pull at equal distances from the center and exert a force of 56 pounds each, what is the distance?

267. Is there any reason why a man should put his shoulder to the spoke of the wheel rather than to the body of the wagon in helping it up hill?

268. A rod AB, of length 15 feet, is supported by props at A and B; a weight of 200 pounds is suspended from the rod at a point 7 feet from A. Find the pressure on the prop at A.

269. A light bar, 9 feet long, to which is attached a weight of 150 pounds, at a point 3 feet from one end, is borne by two men. Find what portion of the weight is borne by each man, when the bar is horizontal.

270. A light rod, 16 inches long, rests on two pegs 9 inches apart, with its center midway between them. The greatest weights, which can be suspended separately from the two ends of the rod without disturbing the equilibrium, are 4 pounds and 5 pounds respectively. There is another weight fixed to the rod. Find that weight and its position.

271. A light rod AB, 20 inches long, rests upon two pegs whose distance apart is equal to half the length of the rod. How must it be placed so that the pressure on the pegs may be equal when weights 2W, 3W, are suspended from A, B, respectively?

272. The horizontal roadway of a bridge is 30 feet long and its weight, 6 tons, may be supposed to act at its middle point, and it rests on similar supports at its ends. What pressure is borne by each of the supports when a carriage weighing 2 tons is one-third of the way across the bridge?

273. "We have a set of hay-scales, and sometimes we have to weigh wagons that are too long to go on them. Can we get the correct weight by weighing one end at a time and then adding the two weights?"

274. A rod, 18 inches long, can turn about one of its ends, and a weight of 5 pounds is fixed to a point 6 inches from the fixed end. Find the force which must be applied at the other end to preserve equilibrium.

275. A straight uniform lever weighing 10 pounds rests on a fulcrum one-third of its length from one end; it is loaded with a weight of 4 pounds at that end. Find what vertical force must act at the other end to keep the lever at rest.

276. A weight of 56 pounds is attached to one end of a uniform bar which is ten feet long, and weighs 20 pounds; the fulcrum is 2 feet from the end to which the weight is attached. What weight must be applied at the other end to balance?

277. AB is a horizontal uniform bar $1\frac{1}{2}$ feet long, and F a point in it 10 inches from A. Suppose that AB is a lever turning on a fulcrum under F, and carrying a weight of 40 pounds at B; weight of lever, 4 pounds. If it is kept horizontal by a fixed pin above the rod, 7 inches from F and 3 inches from A, find the pressure on the fulcrum and on the fixed pin.

278. An ununiform rod, 16 feet long, weighing 4 pounds, balances about a point 4 feet from one end. If, 2 feet from this end, a weight of 10 pounds be hung, what weight must there be hung from the other end so that the rod may balance about its middle point?

279. Six men are to carry an iron rail 60 feet long and weighing 90 pounds per yard; each man sustains one-sixth of the weight. Two men are to lift from one end and the other four by means of a cross-bar. Where must the cross-bar be placed?

280. A rod 2 feet long, with a weight of 7 pounds at its middle point, is placed upon two nails, A and B. AB is horizontal and 7 inches long. Find how far the ends of the rod must extend beyond the nails, if the difference of the pressures on the nails be 5 pounds.

281. A davit is supported by a foot-step A and a collar B, placed 5 feet apart. A boat weighing two tons is about to be lowered, and is hanging 4 feet horizontally from vertical through the foot-step and collar. Determine the forces which must be acting at A and B.

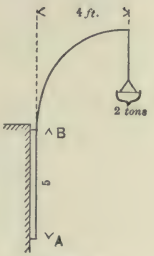


Fig. 33.

282. A highway bridge of span 50 feet, breadth 40 feet, has two queen-post trusses of depth 8 feet; and each truss is divided by two posts into three equal parts. The bridge is designed to carry a load

of 100 pounds per square foot of floor surface. Find the stresses developed.

Find the loads for each truss at the two panel points C and D; then, by the methods of Moments, find the reactions R and R_1, observing, as explained for problem 251, that at each end half a panel of the load goes directly on the abutment and does not affect our computation of stresses in the members of the truss. The reactions thus known makes it possible to find the two unknown forces (stresses in the members) at the abutments. Likewise at foot of posts three forces meet in a point. One is known, — the stress in post which is equal to load at C or D, — and the other two can be found by methods of three forces acting at a point.

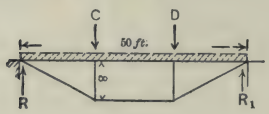


Fig. 34.

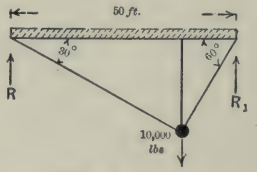


Fig. 35.

283. A king-post truss of 20 feet span, as shown in Fig. 35, has a uniform load of 10 × 200 pounds on the horizontal member and 10 000 pounds at the foot of the post. Determine the reactions and stresses.

284. A 5-foot water-pipe is carried across a gully by two king-post trusses that are spaced 6 feet apart. The pipe when filled with water makes a load of 200 pounds per square foot. Length of trusses is 40 feet ; depth, 5 feet. Find the stresses.

285. A storehouse has queen-post trusses in the top story ; 50 feet span, 10 feet depth, lower chord divided into 3 equal parts ; trusses 8 feet apart, and load 150 pounds per square foot. Find the stresses.

286. A ladder with 21 rungs a foot apart leans against a building with inclination of 45°. Find the pressure against the building when a man weighing 150 pounds stands on the eleventh rung.

287. Like parallel forces of 10 and 20 units act perpendicularly to AB at A and B; a force of 15 units acts from A to B. Find the resultant of the three forces, and show in a diagram how it acts.

288. A rod is acted on at one end by a force of 3 downwards, and at a distance of two feet from this end by a force of 5 upwards. Where must a force of 2 be applied to keep the rod at rest?

289. Three parallel forces of 1 pound each act on a horizontal bar. The right hand one acts vertically upwards, the two others vertically downwards, at distances 2 feet and 3 feet respectively, from the first. Draw their resultant, and state exactly its magnitude and position.

290. A rod is suspended horizontally from points, A and B, 12 feet apart; between A and B points C and D are taken, such that AC = BD = 3 feet; a weight of 120 pounds is hung at C, and a weight of 240 pounds at D; the weight of the rod is neglected. Take a point O, midway between A and B, and find with respect to O the algebraical sum of the moments of the forces acting on the rod on one side of O.

291. A horizontal rod without weight, 6 feet long, rests on two supports at its extremities; a weight of

672 pounds is suspended from the rod at a distance of $2\frac{1}{2}$ feet from one end. Find the reaction at each point of support. If one support could bear a pressure of only 112 pounds, what is the greatest distance from the other support at which the weight could be suspended?

292. Three equal parallel forces act at the corners of an equilateral triangle. Find the point of application of their resultant.

293. Find the center of the three parallel forces 4 pounds, 6, and 8, which act respectively at the corners of an equilateral triangle.

294. P, Q, R, are parallel forces acting in the same direction at the angular points respectively of an equilateral triangle ABC. If $P = 2Q = 3R$, find the position of their center; also find its position if the direction of the force Q is reversed.

295. Show that if two forces be represented in magnitude and direction by two sides of a triangle, taken in order, the sum of their moments about every point in the base is the same.

296. Draw a square whose angular points in order are A, B, C, D, and suppose equal forces (P) to act from D to A, A to B, and B to C respectively, and a fourth force (2P) to act from C to D. Find a point

such that, if the moments of the forces are taken with respect to it, the algebraic sum is zero.

297. ABCD is a square, the length of each side being 4 feet, and four forces act as follows : 2 pounds from D to A, 3 pounds from B to A, 4 pounds from C to B, and 5 pounds from D to B. Find the algebraical sum of the moments of the forces about C.

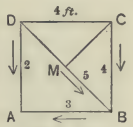


Fig. 36.

The forces act as in the figure.
Draw CM perpendicular to DB.
Then, $CM = DM.$

$$\therefore CD^2 = CM^2 + MD^2 = 2CM^2.$$

$$\therefore CM = \frac{CD}{\sqrt{2}}.$$

$$\therefore CM = \frac{4}{\sqrt{2}} = 2.83 \text{ nearly.}$$

∴ Algebraical sum of the moments about C

$$= -2 \times DC + 3 \times CB + 4 \times 0 - 5 \times CM$$
$$= -2 \times 4 + 3 \times 4 + 0 + 5\ (2.83)$$
$$= -8 + 12 \times 14.15$$
$$= -10.15 \text{ units.}$$

298. ABCD is a square, and AC is a diagonal: forces P, Q, R, act along parallel lines at B, C, D, respectively, Q acts in the direction A to C, P opposite direction, and R in opposite direction. Find, and show in a diagram, the position of the center when $Q = 5P$ and $R = 7P$.

299. Draw a rectangle, ABCD, such that the side AB is three-fourths of the side BC; forces of 3, 9, and 5 units act from B to A, B to C, and D to A respectively. Find their resultant by construction or

otherwise, and show in your diagram exactly how it acts.

300. Prove that, if parallel forces 1, 2, 3, 4, 5, 6, are situated at the angles of a regular hexagon, the distance of their center from the center of the circumscribing circle is two-sevenths of the radius of that circle.

301. Six forces, represented by the sides of a regular hexagon taken in order, act along the sides to turn the hexagon round an axis perpendicular to its plane. Show that the moment of the forces is the same through whatever point within the hexagon the axis passes.

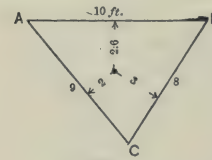


Fig. 37.

302. A triangular table, sides 8 feet, 9 feet, and 10 feet, is supported by legs at each corner, and 350 pounds is placed on it 3 feet from the 8-foot side, 2 feet from the 9-foot side, and 2.6 feet from the 10-foot side. What are the pressures on the legs?

303. A triangular shaped platform right-angled at A, with side AB 10 feet long, side AC 40 feet long, is loaded with freight at 50 pounds per square foot surface. Find the load carried by each of the three corner-posts.

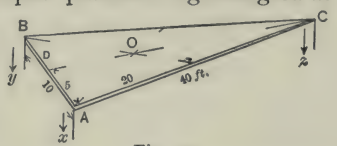


Fig. 38.

O, the center of gravity, is at one-third the distance from the middle of any base to the opposite vertex. Load equals 10 000 pounds.

Take moments about axis AB — thus find load carried by C. Then take moments about sides AC and BC.

304. Four vertical forces, 5, 7, 10, and 12 pounds, act at the corners of a square of 20-inch sides. Find resultant and its point of application.

Let ABCD be the square,

Resultant = 5 + 7 + 10 + 12
= 34 pounds.

To find its point of application:

Resultant of 7 and 10 will be a force of 17 pounds acting from point in line CB distant $\frac{7}{17}$ of 20 inches from B. The resultant of 5 and 12 will be 17 pounds acting at a point in line AD distant $\frac{5}{17}$ of 20 inches from A. The resultant of these two resultants will be a force of 17 + 17 pounds, 34 pounds, acting at a point half way between them, and at a perpendicular distance from AB of

$$\tfrac{1}{2} \text{ of } [\tfrac{7}{17} \times 20 + \tfrac{5}{17} \times 20] = 7\tfrac{1}{17} \text{ inches.}$$

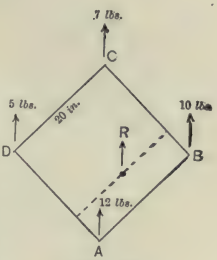


Fig. 39.

305. A floor 20 × 30 feet is supported mainly by four posts, one at each corner. There is a load of 20 pounds per square foot uniformly distributed, and at point O, 5 feet from 30-foot side and 7 feet from 20-foot side, there is a metal planer weighing 5 tons. Find the load on each post.

306. Weights 5, 6, 9, and 7 respectively, are hung from the corners of a horizontal square, 27 inches in a side. Find, by taking moments about two adjacent edges of the square, the point where a single force must be applied to balance the effect of the forces at the corners.

307. A uniform beam, weighing 400 pounds, is suspended by means of two chains fastened one at each end of the beam. When the beam is at rest it is found that the chains make angles of 100° and 115° with the beam. Find the tensions in the chains.

308. A force of 50 pounds acts eastward and a force of 50 pounds acts westward. Will there be motion?

That depends, as will easily be seen, upon the position of the forces. If they act on the two ends of a rope there will be no motion. If they act one on the northerly part of a brake wheel and one on the southerly part there will be motion, — that of rotation.

Such forces produce a

COUPLE: two equal, opposite, parallel forces not acting in the same straight line.

The tendency to motion by couples is not of translation but of rotation. The measure of this tendency is, —

Moment of a couple equals the product of one of the two forces × perpendicular distance between them.

What is the resultant of a couple of moment 15, and a force 3?

309. A brakeman sets up a brake on a freight car by pulling 50 pounds with one hand and pushing 50 pounds with the other; his forces act tangentially to the brake wheel, the diameter of which is $1\frac{1}{2}$ feet. Another time he produces the same brake resistance by using a lever in handwheel and pulling 25 pounds. How far from handwheel must his hands be placed?

310. When are couples said to be like and when unlike? When will two unlike couples balance each

other? (1) If a system of forces is represented in magnitude and position by the sides of a plane polygon taken in order, show that the system must be equivalent to a couple. (2) If the sides of a parallelogram taken in order represent a system of forces acting upon a body, express the moment of the couple to which the system of forces is equivalent.

311. Show that a force and a couple in one plane may be reduced to a single force. Given in position a force of 10 pounds, and a couple consisting of two forces of 4 pounds each, at a distance of 2 inches, acting with the hands of a clock, draw the equivalent single force.

312. The length of the side of a square ABCD is 12 inches. Along the sides AB and CD forces of 10 pounds act, and along AD, CB forces of 20 pounds. Find the moment of the equivalent couple.

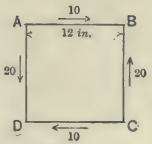


Fig. 40.

Moments about D,

$-12 \times 10 + 12 \times 20 =$ moment of equivalent-couple

$12 \times 10 =$ moment of equivalent-couple

313. Forces P and Q act at A, and are completely represented by AB and AC, sides of a triangle ABC. Find a third force R such that the three forces together may be equivalent to a couple whose moment is represented by half the area of the triangle.

314. A tradesman has a balance with arms of unequal length, but tries to be fair by weighing his ma-

terial first from one scale pan, then from the other. Show that he will defraud himself.

315. A tradesman uses a balance with arms in ratio of 5 to 6; he weighs out from alternate pans what appears to be 30 pounds. How much does he gain or lose?

316. The beam of a balance is 6 feet long, and it appears correct when empty; a certain body placed in one scale weighs 120 pounds, when placed in the other, 121 pounds. Show that the fulcrum must be distant about $\frac{1}{13}$ of an inch from the center of the beam.

317. The weight of a steelyard is 12 pounds, its movable weight is 3 pounds. Find the distance between successive pound graduations, if the length of the short arm is 3 inches.

318. A weight of 247 pounds is attached to one end of a horizontal straight lever, which is 22 inches long, and may be regarded as having no weight; the force is applied at the other end, and makes an angle of 27° with the lever; the fulcrum is 3 inches from the weight. Find the magnitude of the force when it just balances the weight.

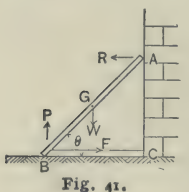


Fig. 41.

319. A uniform beam rests at a given inclination, θ, with one end against a smooth vertical wall, and the other end on smooth horizontal ground: it is held from slipping by a string extending horizontally from

the foot of the beam to the foot of the wall. Find the tension in the string and the pressures at the ground and wall.

AB is the beam, AC the wall, BC the string, W the weight of the beam acting at its middle point G.

There are three forces supporting the beam: vertical reaction P, horizontal reaction R, and tension in the string F.

Take moments about B, the point of intersection of two of the forces — their lever arms would be zero.

$$R \times AC = W \times \frac{BC}{2}.$$

Substitute for AC its value $BC \times \tan \theta$, then

$$(1)\ R = \frac{W}{2 \tan \theta}$$

but R must equal F, both being horizontal resisting forces that maintain equilibrium; likewise P and W must be equal.

$$\therefore (2)\ F = \frac{W}{2 \tan \theta} \text{ and}$$

$$(3)\ P = W$$

320. A uniform beam rests with a smooth end against the junction of the horizontal ground and a vertical wall; it is supported by a string fastened to the other end of the beam and to a staple in the vertical wall. Find the tension of the string, and show that it will be half the weight of the beam if the length of the string be equal to the height of the staple above the ground.

321. A uniform rod 8 feet long, weighing 18 pounds, is fastened at one end to a vertical wall by a smooth hinge, and is free to move in a vertical plane perpendicular to the wall. It is kept horizontal by a string 10 feet long, attached to its free end and to a

point in the wall. Find the tension in the string, and the pressure on the hinge.

322. A uniform beam, 12 feet in length, rests with one end against the base of a wall which is 20 feet high. If the beam be held by a rope 13 feet long, attached to the top of the beam and to the summit of the wall, find the tension of the rope, neglecting its weight, and assuming the weight of the beam to be 100 pounds.

323. ABC is a rigid equilateral triangle, weight not considered; the vertex B is fastened by a hinge to a wall, while the vertex C rests against the wall under B. If a given weight is hung from A, find the reactions at B and C. What are the magnitudes and directions of the forces exerted by the weight on the wall at B and C?

324. A beam AB rests on the smooth ground at A and on a smooth inclined plane at B; a string is fastened at B and, passing over a smooth peg at the top of the plane, supports a weight P. If W is the weight of the beam, and α the inclination of the plane, find P and the reactions on the rod.

Draw the figure.

The weight W acts at the middle point C. The reaction of the ground at A is R, upwards.

The reaction of the plane at B is R_1, perpendicular to the plane.

Let the angle BAD $= \theta$.

The tension of the string at B = tension of the string throughout = P.

There are four forces acting on the beam, W, R, R_1, P.

Resolve vertically and horizontally.

325. A pole 12 feet long, weighing 25 pounds, rests with one end against the foot of a wall, and from a point 2 feet from the other end a cord runs horizontally to a point in the wall 8 feet from the ground. Find the tension of the cord and the pressure of the lower end of the pole.

326. A light smooth stick 3 feet long is loaded at one end with 8 ounces of lead; the other end rests against a smooth vertical wall, and across a nail which is 1 foot from the wall. Find the position of equilibrium and the pressure on the nail and on the wall.

327. A trapezoidal wall has a vertical back and a sloping front face; width of base, 10 feet; width of top, 7 feet; height, 30 feet. What horizontal force must be applied at a point 20 feet from the top in order to overturn it? Thickness of wall, 1 foot; weight of masonry in wall, 130 pounds per cubic foot.

328. Six men using a rope 50 feet long were just able to pull over a chimney 75 feet high. How far up from the bottom of the chimney was it advisable to attach the rope?

329. If 150 000 pounds is the thrust along the connecting rod of the engine, in example 86, $2\frac{1}{2}$ feet the crank radius, and the connecting-rod is inclined to the crank axis at 150°, show that the moment of the thrust about the crank-pin is one-half the greatest possible moment.

330. A trap-door of uniform thickness, 5 feet long and 3 feet wide, and weighing 5 hundred weight, is

held open at angle of 35° with the horizontal by means of a chain. One end of chain is hooked at middle of top edge of door, and the other is fastened at wall 4 feet above hinges. Find the force in the chain and the force at each hinge.

331. The sketch represents a coal wagon weighing with its load $4\frac{1}{2}$ tons. How many pounds applied at P by usual methods of hand power will just lift the wagon when in the position shown in the sketch?

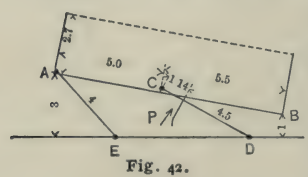


Fig. 42.

AE is a rod in tension. CD is a connecting-bar. Divide the problem into three parts:

(*a*) Draw the forces acting.

(*b*) Find horizontal distance from C to the vertical through the center of gravity.

(*c*) Find force to apply at C parallel to P; then find P.

CENTER OF GRAVITY

332. A rod of uniform section and density, weighing 3 pounds, rests on two points, one under each end; a movable weight of 4 pounds is placed on the rod. Where must it be placed so that one of the points may sustain a pressure of 3 pounds, and the other a pressure of 4 pounds?

333. Two rods of uniform density weighing 2 pounds and 3 pounds respectively are put together so that the 3-pound one stands on the middle of the other. Find the center of gravity of the whole.

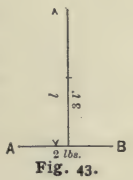


Fig. 43.

Take moments about AB,

$$+3 \times \tfrac{1}{2}l - 5 \times x = 0$$

334. A thin plate of metal is in the shape of a square and equilateral triangle, having one side common; the side of the square is 12 inches long. Find the center of gravity of the plate.

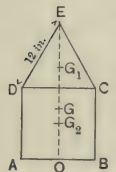


Fig. 44.

Let G_1 be the center of gravity of the triangle, G_2 of the square, G of the whole plate.

From symmetry EG_1GG_2O will be a straight line bisecting the plate, and

$$OG_2 = 6 \text{ inches}$$

$$OG_1 = 15.5 \text{ inches}$$

Let w = weight of metal per square inch

$$\text{Area of triangle} = \tfrac{1}{2} \times 12 \times \sqrt{12^2 - 6^2}$$

$$= 62.4 \text{ square inches}$$

$$\text{Weight} = 62.4 \text{ pounds} \times w \text{ pounds}$$

$$\text{Area of square} = 144 \text{ square inches}$$

$$\text{Weight} = 144 \times w \text{ pounds}$$

Take moments about the axis AB,

$$\text{Weight of triangle} \times OG_1 + \text{weight of square} \times OG_2 - \text{total weight} \times OG = 0$$

$$62.4w \times 15.5 + 144w \times 6 - (62.4w + 144w) \times OG = 0$$

$$\therefore OG = 8.86 \text{ inches.}$$

335. A bridge member has two web plates 18 × $\frac{3}{4}$ inches, top plate 21 × $\frac{3}{8}$, top angles 3 × 3 and $\frac{3}{8}$ inches thick, bottom angles 4 × 3 and $\frac{7}{16}$ inches thick. Find "eccentricity" — the distance from AB, the neutral axis through the center of gravity to C, the middle of the section.

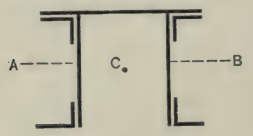


Fig. 45.

Fig. 46.

336. Web plate of Fig. 46 is 10 × $\frac{1}{2}$ inches, top plate 12 × $\frac{1}{2}$, two angles 4 × 3 × $\frac{3}{8}$. Find "eccentricity." (Given in Osborn's Tables (1905) page 24.)

337. Fig. 47 shows a cross-section of the top chord of one of the main trusses in the Portage Canal Draw-Bridge at Houghton, Mich. See Engineering News of June 15, 1905. In computing the strength of this built-up member, it is required to find the position of the axis AB that passes through the center of gravity of the section.

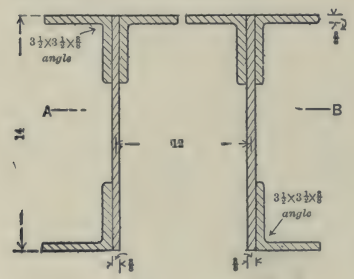


Fig. 47.

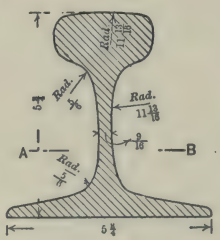


Fig. 48.

338. The strength of steel rails is usually computed by embodying, among other factors, the distance from neutral axis, which passes through the center of gravity, to the extreme fibres of the section. A 100-pound rail, of the Lorain Steel Company, has a section shown in Fig. 48. Draw the section carefully to full scale on bristol board; then cut it out and locate its center of gravity by balancing on a knife edge. What is the distance from center of gravity to extreme fibres?

339. ABC is a triangle with a right angle at A. AB = 3 inches; AC = 4 inches; weights of 2 ounces, 3 and 4, are placed at A, B, and C. Find the position of their center of gravity.

340. A uniform triangle ABC of weight W, and lying on a horizontal table, is just raised by a vertical force applied at A. Find the magnitude of this force, and that of the resultant pressure between the base BC and the table.

341. A uniform circular disk has a circular hole punched out of it, extending from the circumference half way to the center. Find the center of gravity of the remainder.

342. A box, including its cover, is made of six equal square boards; where is its center of gravity when its lid is turned back through an angle of 180°?

343. ABCD is a thin rectangular plate weighing 50 pounds, AB is 10 feet, BC 2 feet; the plate is suspended by the middle point of its upper edge AB, and then, of course, AB is horizontal, but if a weight of 5 pounds is placed at A, AB will become inclined to the horizon. Show how to find the angle of inclination either by calculation or by construction.

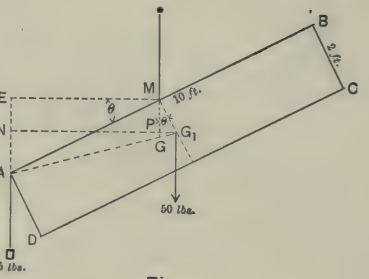


Fig. 49.

344. A circular disk, 8 inches in diameter, has a hole 2 inches in diameter punched out of it, the center of the hole being 3 inches from the circumference of the disk. Find the center of gravity of the remaining portion.

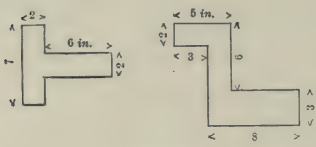


Fig. 50.

345. Find the centers of area of the above sections of uniform plate metals.

346. Into a hollow cylindrical vessel 11 inches high and weighing 10 pounds, the center of gravity of which is 5 inches from the base, a uniform solid cylinder 6 inches long and weighing 20 pounds is just fitted. Find the common center of gravity.

G_1 center of gravity of hollow cylinder

G_2 center of gravity of solid cylinder.

Moments about AB,

$$+ 10 \times 5 + 20 \times 3 - 30 \times x = 0$$

$$+ 50 + 60 - 30x = 0$$

$$30x = 110$$

$$x = 3\tfrac{2}{3} \text{ inches.}$$

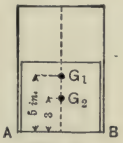


Fig. 51.

347. Give examples of stable and unstable equilibrium. A cone and a hemisphere of the same material are cemented together at the common circular base. If they are on a horizontal plane, and the hemisphere in contact with the plane, find the height of the cone in order that the equilibrium may be neutral. (The center of gravity of a hemisphere divides a radius in the ratio of 3 to 5.)

348. A thread 9 feet long has its ends fastened to the ends of a rod 6 feet long; the rod is supported in such a manner as to be capable of turning freely round a point 2 feet from one end; a weight is placed on the thread, like a bead on a string. Find the position in which the rod will come to rest, it being supposed that the rod is without weight, and that there is no friction between the weight and the thread.

349. A circular disk weighs 9 ounces; a thin straight wire as long as the radius of the circle weighs 7 ounces; if the wire is placed on the disk so as to be a chord of the circle, the center of gravity of the whole will be at a distance from the center of the circle equal to some fractional part of the radius. Find that fraction by construction or calculation.

350. A cone and a hemisphere are on the same base. What height must the cone be in order that the center of gravity of the whole solid shall be at the center of the common base?

r = radius common base.
h = height of cone.

FRICTION

The coefficients of friction for various pairs of substances have been found experimentally by Morin; these results however can be used only for approximate computation; actual trial should be made for specific cases. Average values are:

Stone on stone	0.40 to 0.65
Wood on wood	0.25 to 0.40
Metal on metal, dry	0.15 to 0.30
well oiled . . .	0.01 to 0.10

1. Friction is proportional to normal reaction, R.
2. Is independent of area of contact.
3. Is dependent very much on the roughness of surfaces.

351. Define "coefficient" and "angle of friction," and "resultant reaction."

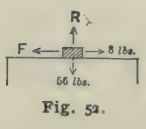


Fig. 52.

352. A weight of 56 pounds is moved along a horizontal table by a force of 8 pounds. How much is the coefficient of friction?

The pull of 8 pounds is required to overcome friction, and is equal to the friction.

Friction = coefficient × Reaction (perpendicular to plane of table.

$$F = \mu \times R$$
$$= \mu \times 56 \text{ pounds}$$
$$8 = \mu \times 56$$
$$u = \tfrac{8}{56}$$
$$= \tfrac{1}{7}.$$

353. A 5 × 8-foot vertical gate has a head of water against its center equal to 10 feet, or $4\frac{1}{3}$ pounds per square inch. The coefficient of friction being 0.40, what force is required in raising it to overome the friction?

354. A horizontal pull of 50 pounds is required to slide a trunk along the floor. The coefficient of friction is 0.20, and trunk when empty weighs 75 pounds. How many pounds of goods does it contain?

355. A block of stone is dragged along the ground by a horse exerting a force of 224 pounds. If $\mu = 0.6$, what is the weight of the block?

356. A weight of 500 pounds is placed on a table, and can hardly be slid by a horizontal pull of 155 pounds. Find the coefficient of friction, and the number of degrees in the angle of friction by measuring from a drawing made to a scale.

357. A stone just slides down a hill of inclination 30°. What is the coefficient of friction?

358. A block rests on a plane which is tilted till the block commences to slide. The inclination is found to be 8.4 inches at starting, and afterwards 6.3 inches on a horizontal length of 2 feet. Find the co-

efficient of friction when the block starts to slide, and after it has started.

359. A horse draws a load weighing 2 000 pounds up a grade of 1 in 20; the resistance on the level is 100 pounds per ton. Find the pull on the traces when they are parallel with the incline.

360. How much work has a man, weighing 224 pounds, done in walking twenty miles up a slope of 1 vertical to 40 horizontal? What force could drag a dead load of the same weight up the same hill (*a*) if friction be negligible, (*b*) if friction be $\frac{1}{4}$ of the weight?

361. Three artillerymen drag a gun weighing 1 700 pounds up a hill rising 2 vertically in 17 horizontally. Suppose the resistance to the wheels going up the hill be 16 pounds per hundred weight, what pull parallel to the hill must each exert to move it?

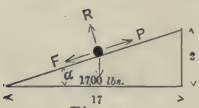


Fig. 53.

When the gun is about to move forward the pull P will be acting up the plane, and parallel to it; the friction F down the plane, holding back; the force R perpendicular to inclined plane, partly supporting the gun, and W the weight of the gun acting vertically downward. Weight of gun is given — 1 700 pounds. Resolve into components perpendicular and parallel to the plane. The perpendicular component will be the supporting force of the plane — its reaction R; the parallel component will be the part of the pull P required by weight of the gun.

362. Find the force which, acting in a given direction, will just support a body of given weight on a

rough inclined plane. The height is to the base of the plane as 3 to 4, and it is found that the body is just supported on it by a horizontal force equal to half the weight of the body. Find the coefficient of friction between the body and the plane.

363. The table of a small planing-machine which weighs 112 pounds makes six single strokes of $4\frac{1}{2}$ feet each per minute. The coefficient of friction between the sliding surfaces is .07. What is the work in foot-pounds per minute performed in moving the table?

364. A rectangular block ABCD whose height is double its base, stands with its base AD on a rough floor, coefficient of friction $\frac{1}{6}$. If it be pulled by a horizontal force at C till motion ensues, determine whether it will slip on the floor, or begin to turn over round D.

365. A cubical block rests on a rough plank with its edges parallel to the edges of the plank. If, as the plank is gradually raised, the block turns over on it before slipping, how much at least must be the coefficient of friction?

366. A weight of 5 pounds can just be supported on a rough inclined plane by a weight of 2 pounds, or can just support a weight of 4 pounds suspended by a string passing over a smooth pulley at the vertex. Find the coefficient of friction, and the inclination of the plane.

367. Find the least force that will drag a box weighing 200 pounds along a concrete floor, the coefficient of friction being 0.50.

The required force will of course not act horizontally, but instead in some direction as P. To find the angle b:

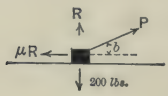


Fig. 54.

Resolve vertically

$$-P \sin b - R + 200 = 0$$

Resolve horizontally

$$+P \cos b - \mu R = 0$$

From these two equations

$$P = \frac{200\mu}{\mu \sin b + \cos b}.$$

When will P be as small as possible? When $\mu \sin b + \cos b$ is as large as possible. The student not familiar with the calculus can find by trial that the maximum value of denominator, or least value of the pull P, will occur when $b = \tan^{-1}\mu$, that is, the angle whose tangent is μ.

By the method of calculus,

$$\mu \sin b + \cos b = x$$

Differentiate, noting that μ is a constant; and, to find a critical value, which in this case will be a maximum value, place the first differential equal to zero.

$$\frac{dx}{db} = \mu \cos b - \sin b = 0$$

$$\mu = \tan b$$

$$b = \tan^{-1}\mu$$

$$= 26° 34'$$

$$P = \frac{200 \times \frac{1}{2}}{\frac{1}{2} \times .447 + .894}$$

$$= 89 \text{ pounds.}$$

368. By experiment it was found that a box of sand weighing 204 pounds required a least pull of 110 pounds (at angle a) to move it on a concrete floor. What was the value of μ?

369. The roughness of a plane of inclination 30° is such that a body of weight 500 pounds can just rest on it. What is the least force required to draw the body up the plane?

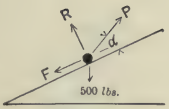


Fig. 55.

As in problem 367 a will equal the angle of friction, or $\tan^{-1}\mu$.

370. A sled of total weight 3 tons is to be drawn up a grade of 1 vertical to 8 horizontal. The coefficient of friction between the sled shoes and the snow is 0.10. What angle should the traces make with the horizontal? What pull will the horse exert?

The problems that pertain to the wedge can be solved by the same methods that have been used for the inclined plane. The essential principles are: Show the conditions by a sketch, indicating carefully the position and direction of all forces; then, (1) resolve parallel to plane. (2) resolve perpendicular to plane. Thus for problem 371

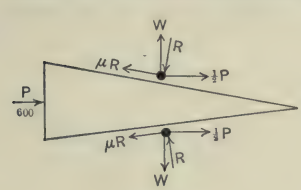


Fig. 56.

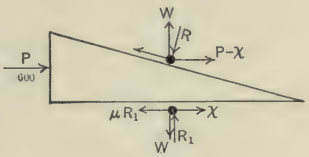


Fig. 57.

Fig. 56 shows the conditions for one form of wedge, Fig. 57 for another.

Observe the directions of R and W. As can be seen in Fig. 56, ½ P should decrease the value of R, therefore R must act in the direction indicated. In previous problems the weight has moved on the inclined plane; here the plane moves.

Resolve || to top plane (Fig. 56),

$$-0.20\,R + 300\cos 3°\,35' - W \times \sin 3°\,35' = 0.$$

Resolve ⊥ to plane,

$$+ R - 300 \sin 3° 35' - W \times \cos 3° 35' = 0.$$

Solve these two equations for W.

To find the pull necessary to withdraw the wedge, sketch another figure showing μR and $\frac{1}{2} P$ in their new positions. Then solve as indicated above.

371. A cotter, or wedge, having a taper of 1 in 8, is driven into a cottered joint with an estimated pressure of 600 pounds. Taking the coefficient of friction between the two surfaces as 0.2, find the force which the wedge exerts at the joint perpendicular to the pressure of 600 pounds; also find the pull necessary to withdraw the wedge.

372. A floor-column with its load of 5 tons is to be lifted by two wedges driven towards each other. Thickness of each wedge is 2 inches, length, 12 inches; coefficient of friction, 0.15. Find the force that must be equivalent to P in order to drive the wedge.

373. A casting of weight 5 000 pounds is to be lifted by an iron wedge that is forced ahead by a screw and mechanism that can give an equivalent force of 3 000 pounds. If a 12-inch wedge is used, what should be its thickness?

374. A rough wedge has been inserted into a block and is only acted on by the reactions. If it is on the point of slipping out, and the coefficient of friction is $\frac{1}{\sqrt{3}}$, what is the angle of the wedge?

375. A steel wedge 12 inches long, 2 inches thick, tapering on both sides to 0, is used to wedge up a pump plunger weighing 3 000 pounds by means of a maul weighing 5 pounds. The coefficient of friction is 0.15 and the striking velocity of the maul is 25 feet per second. How far will each blow drive the wedge?

376. A wheel of weight W rests between two planes, each inclined to the vertical at angle α; the plane of the wheel is perpendicular to the line of intersection of the two planes, which is itself horizontal. If μ be the coefficient of friction, find the least couple necessary to turn the wheel.

377. A ladder inclined at an angle of 60° to the horizon rests with one end on rough pavement, and the other end against a smooth vertical wall; the ladder begins to slide down when a weight is put at its middle point. Show that the coefficient of friction is $\frac{\sqrt{3}}{6}$.

When the ladder begins to slide down, the limiting friction would be μR.

Resolve vertically,

$$W = R$$

Resolve horizontally,

$$R' = \mu R$$

Take moments about B, and then solve for μ.

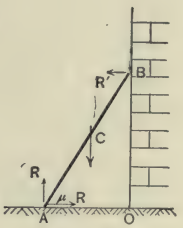


Fig. 58.

If the wall should be rough there would be acting at B an upward force of $\mu' R'$ that would have to be embodied in the above equations.

378. A uniform ladder weighing 100 pounds and 52 feet long is inclined at an angle of 45° with a rough vertical wall and a rough horizontal plane. If the coefficient of friction is at each end $\frac{2}{3}$, how far up the ladder can a man weighing 200 pounds ascend before the ladder begins to slip?

379. A uniform ladder 30 feet long is equally inclined to a vertical wall and the horizontal ground, both rough; a man with a hod — weight 224 pounds — ascends the ladder which weighs 200 pounds. How far up the ladder can the man ascend before it slips, the tangent of the angle of resistance for the wall being $\frac{1}{3}$ and for the ground $\frac{1}{2}$?

380. A uniform beam rests with one end on a rough horizontal plane, and the other against a rough vertical wall, and when inclined to the horizon at an angle of 30° is on the point of slipping down; suppose the surfaces equally rough, find μ.

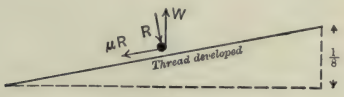
381. A bolt for a cylinder head has 8 threads per inch; mean diameter of threads $1\frac{1}{2}$ inches, average outside diameter of nut $2\frac{3}{4}$ inches, inside diameter of bearing surface, 1.6 inches. The nut is to be tightened by a pull on the end of a 3-foot wrench. The coefficient of friction for threads and underneath the nut being 0.15, what pull should be exerted in order that the stress in the bolt shall not exceed 50 000 pounds?

Problems pertaining to bolt and nut friction can be solved by applying the combined principles of Work and Friction. Thus for

the above problem suppose that the specified conditions should exist for one revolution. This involves no approximation, simply a convenience in numerical figures which otherwise would have to be divided by perhaps a hundred or thousand to apply to a fractional part of a revolution. Then,

$$\underset{\text{on wrench}}{\text{Work}} = \underset{\text{on threads}}{\text{Work}} + \underset{\text{under nut}}{\text{Work}} + \underset{\text{of lifting}}{\text{Work}}$$

The values of work on threads and work under nut can be determined near enough for ordinary cases by slight approximations. As shown by Fig. 59, W = .99 R, or usually R may be taken equal to W also the length of thread developed (for one revolution) $\pi \times 1\frac{1}{2}''$; and in Fig. 60 the circumference C is one that can be determined by the condition that the work done by the friction of all the particles outside is the same as that done by all the particles inside. Its radius for a section like Fig. 60 is $x = \frac{\frac{2}{3}(R^3 - r^3)}{(R^2 - r^2)}$, or, for this case, $x = 1.11$ inches, which is approximately the same as would



As the thread advances it acts like a wedge.

Fig. 59.

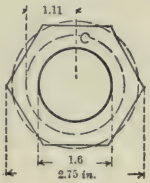


Fig. 60.

result by taking a mean circumference between 1.6 inches diameter and 2.75 — or 1.09 inches. Therefore ordinarily use the mean circumference for the position of friction under nut.

The equation for Work thus becomes:

For one revolution,

$$\underset{\text{Wrench}}{P}(3 \times 12 \times 2 \times 3\tfrac{1}{7}) = \underset{\text{Threads}}{50\,000 \times 0.15 \times 4.71} + \underset{\text{Nut}}{50\,000 \times 0.15 \times 1.09 \times 2 \times 3\tfrac{1}{7}} + \underset{\text{Lifting}}{50\,000 \times 0.125}.$$

From which equation find P, the pull that should be exerted on wrench to produce 50 000 pounds stress in the bolt.

382. By trial in a 60 000-pound testing machine we have obtained with a builder's lifting-jack a stress on the machine of 6 000 pounds for a certain pull on the end of an 18-inch bar. What was that pull? Mean diameter of threads was 1.50 inches, there were 3 threads to the inch, and diameter of bearing that corresponds to the mean circumference of nut described under problem 381 was 1.78 inches. Coefficient of friction for threads was 0.15, and for bearing 0.15.

383. A locomotive bolt has 10 threads to the inch; mean diameter 2 inches, average outside diameter of nut $4\frac{1}{2}$ inches, diameter of hole in washer on which nut turns 2.2 inches. If length of wrench was 5 feet, pull 367 pounds, and stress 40 000 pounds, what was the value of the coefficient of friction?

384. A test of rope friction in our engineering laboratory at Tufts College has given the following result:

(The weight T_1 just moving, and pull T_2 resisting any increased motion. See Fig. 61.)

FOR WEIGHT OF T_1 = 100 POUNDS.

Number of Laps	Pull T_2 Lbs.	Ratio $\frac{T_1}{T_2}$	Number of Laps	Pull T_2 Lbs.	Ratio $\frac{T_1}{T_2}$
$\frac{1}{4}$	81	1.23	$1\frac{3}{4}$	14	
$\frac{1}{2}$	65	1.54	2	11	
$\frac{3}{4}$	45		$2\frac{1}{4}$	8	
1	32		$2\frac{1}{2}$	5	
$1\frac{1}{4}$	25		$2\frac{3}{4}$	$4\frac{1}{2}$	
$1\frac{1}{2}$	19		3	$3\frac{1}{2}$	

Compute the ratios of T_1 and T_2, then plot the results, using a scale of 1 inch = 1 lap for vertical ordinates, and 1 inch = ratio of

10 for horizontal. Sketch the most probable curve for the plotted points, observing that it does not necessarily pass through the last point, and determine whether or not it should pass through the origin.

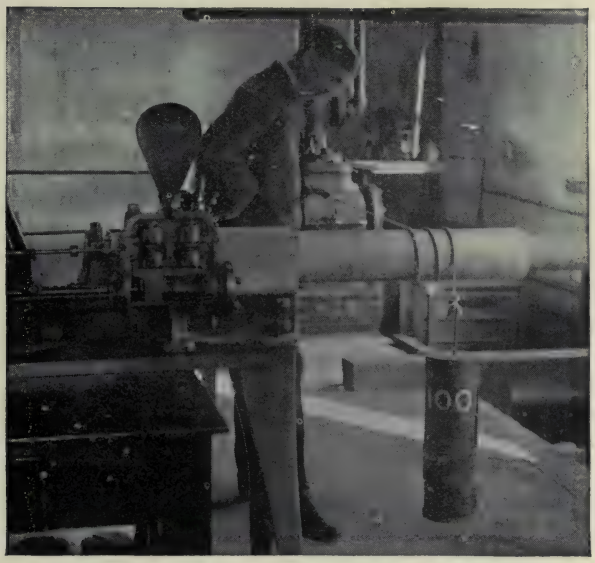
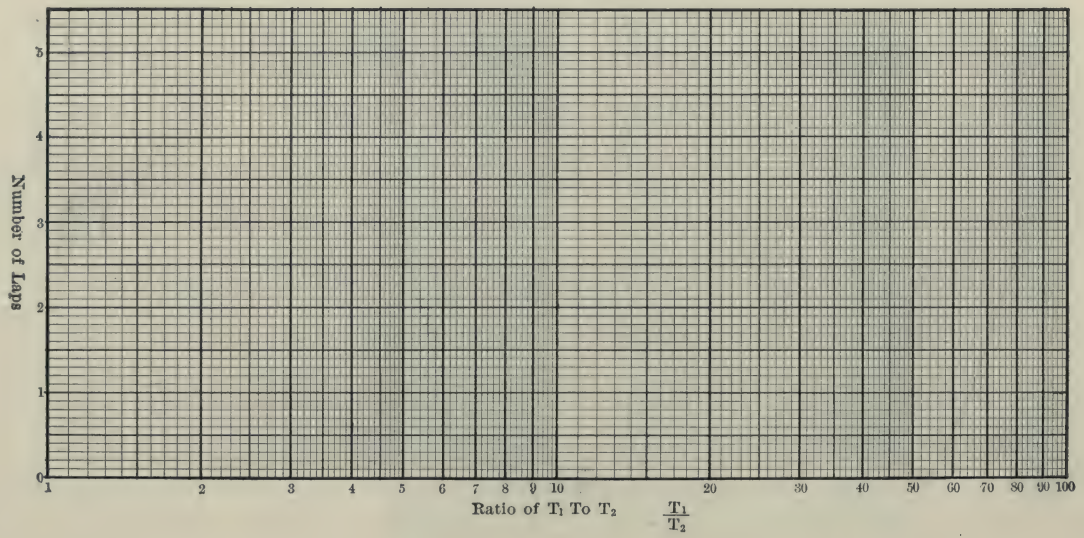


Fig. 61.

385. Now plot the same results on the specially ruled paper of page 108.

(This form of ruling was used by the Burr–Hering–Freeman Commission on Additional Water Supply of the city of New York (1903) for plotting wide ranges of values in a small space [7 million to 40 million in a 1½-inch space], yet affording increased scale for the small values. It has not, to my knowledge, been used before for mechanics' problems of this sort.)


Number of Laps
0
1
2
3
4
5
1
2
3
4
5
6
7
8
9
10
20
30
40
50
60
70
80
90
100
Ratio of T_1 To T_2 $\frac{T_1}{T_2}$

Determine how this form of ruling is constructed. Plot the points for laps and ratios, and draw the most probable straight line through the points as before. Should the line pass through the origin? How do the points for $1\frac{1}{2}$ laps and $2\frac{3}{4}$ compare on this straight line with those on the curved line previously plotted?

386. The lines plotted for the preceding problems are sufficient to answer directly many questions pertaining to that particular rope and piece of timber. For the same conditions, how many laps are needed to hold a weight of 300 pounds with a pull of 40 pounds?

387. For the same conditions, with 2 laps and a pull of 100 pounds, what weight could be lowered into the hold of a vessel?

388. It is evident that the plotted lines of the preceding problems would not apply to other cases of friction. The value of μ the coefficient of friction is contained in the equation of the curves, but cannot yet be specified.

For the purpose of finding the value of the coefficient, as will be done later on, it is advisable to determine the slope and position of the plotted line; that is, its equation. Notice that n (the number of laps) = c (a constant depending on the slope of the line) $\times \log \frac{T_1}{T_2}$ (the ratio of the two tensions). The value of c (the slope of the line) would depend upon the stiffness of rope and roughness of the rubbing surfaces. For this particular rope and piece of wood, the value of c, according to one plotting that I have, is 2.08.

What does your plotting indicate for the above experiment?

389. From the above the equation of the line becomes $n = 2.08 \log \frac{T_1}{T_2}$. Write your equation and transpose so as to write the value of $\log T_1$, which I find to be $\log T_1 = \log T_2 + .481 \times n$. Now this equation derived from the laboratory experiment will be seen to bear a close relation to the general formula for rope and belt friction which will now be developed.

The author's method of analysis is introduced here for the reason that he believes it to be more easily understood than the methods usually presented in text books.

At B there is a tension of T_2, at A, T_1, and at ends of any small arc C, there are two tensions, T and $T + d'T$. Now if we knew the reaction at the arc C we could multiply it by μ and obtain the

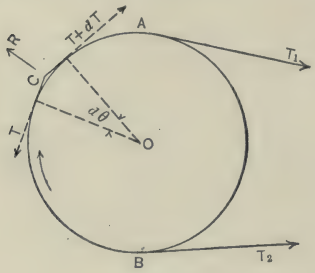


Fig. 62.

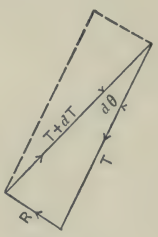


Fig. 63.

friction. To find this reaction R draw the parallelogram of force, Fig. 63. Let an arc of the angle $d\theta$ measured at unit distance from the center be arc $d\theta$, then at distance T the arc would be T × arc $d\theta$, and for a very small angle — a differential angle — this value of the arc would equal R.

$$R = T \text{ arc } d\theta$$
$$\mu R = \mu T \text{ arc } d\theta.$$

Now the friction μR = the difference in tensions dT.

$$dT = \mu T \text{ arc } d\theta.$$

That is, for an infinitesimal arc, the difference in tension $= \mu T \times$ the infinitesimal arc. If we take a very large number of small arcs we can find the friction at each point, add up and get the sum total of friction; or, summing up by the calculus,

$$\int_{T_2}^{T_1} \frac{dT}{T} = \mu \int_0^{\theta} \text{arc } d\theta$$

$$\log_e T_1 - \log_e T_2 = \mu\theta$$

Now $\theta = 2\pi n$ in which n is the number of laps; and the Naperian log can be changed to the common system by multiplying by .4343.

$$\therefore \log_{10} T_1 - \log_{10} T_2 = .4343\,\mu \times 2\pi n$$

$$\log T_1 = \log T_2 + 2.7288\,\mu n$$

which is the general formula for rope and belt friction. It contains variable quantities: the tension T_1, tension T_2, and the number of laps n. Any two of these being given the third can be found.

The formula deduced by experiment in problem 389 was

$$\log T_1 = \log T_2 + .481\,n.$$

The similarity of this with the general formula is evident. The last term must contain the value of μ the coefficient of friction. To find this value solve the two equations, and we have

$$2.7288\,\mu n = .481\,n$$

$$\mu = 0.18$$

Then if this be taken as the value of μ, how many laps would be necessary, according to the general formula, for 100 pounds to just move 14 pounds? (Check result with data of problem 384.)

390. A weight of 100 pounds just moves 37 pounds, both being connected by a plain leather belt that encircles one-half of a 14-inch iron pulley that does not turn. Plot the point on the paper of page 108; draw the line of friction, and write the equation of the line. Then compare with the general formula

and determine the value of the coefficient of friction for the plain belt and iron pulley.

Fig. 64.

391. In the same way as by problem 390, after the belt had been treated with "cling-fast" belt dressing 55 pounds just moved 13. Plot the line and find the coefficient of friction.

Further application of the general belt and rope friction formula is seen in the problems that follow.

392. According to conditions of friction as in problem 390, how many turns would have to be taken around a capstan in order to lower a barrel of salt, 150 pounds, into a dory without pulling over 50 pounds?

393. A weight of 5 tons is to be raised from the hold of a steamer by means of a rope which takes $3\frac{1}{2}$ turns around the drum of a steam-windlass. If $\mu = 0.234$, what force must a man exert on the other end of the rope?

394. A man by taking $2\frac{1}{2}$ turns around a post with a rope, and holding back with a force of 200 pounds, just keeps the rope from surging. Supposing $\mu = 0.168$, find the tension at the other end of the rope.

395. A leather belt will stand a pull of 200 pounds. It passes around one-half the circumference of a pulley that is 4 feet in diameter and making 150 revolutions per minute. What power will it transmit if the coefficient of friction between the belt and pulley is 0.1 ?

396. A belt laps 150° around a 3-foot pulley, making 130 revolutions per minute; the coefficient of friction is 0.35. What is the maximum pull on the belt when 20 horse-power is being transmitted and the belt is just on the point of slipping?

397. A weight of 2 000 pounds is to be lowered into the hold of a ship by means of a rope which passes over and around a spar lashed across the hatch-coamings so as to have an arc of contact of $1\frac{1}{4}$ circumferences. If $\mu = \frac{7}{22}$, what force must a man exert at the end of the rope to control the weight?

398. A hawser is subjected to a stress of 10 000 pounds. How many turns must be taken around the bitts, in order that a man who cannot pull more than

250 pounds may keep it from surging, supposing $\mu = 0.168$?

399. A rope drive carrying 20 ropes has a pulley 16 feet in diameter, and transmits 600 horse-power when running at 90 revolutions per minute. Taking $\mu = 0.7$ and the angle of contact 180°, find the tensions on the tight and slack sides of the ropes.

From the data that is given find by the principles of Work the *force* that each rope is transmitting. It is

$$T_1 - T_2 = 218.8 \text{ pounds}$$

Substitute this value of T_2 in the general formula, also the value of μ and n; then

$$\log T_1 = \log (T_1 - 218.8) + .9550$$

$$\log \left(\frac{T_1}{T_1 - 218.8} \right) = .955$$

$$\frac{T_1}{T_1 - 218.8} = 9.02$$

From which find T_1; then find T_2.

400. A belt for a dynamo is to encircle half of an 18-inch pulley. The speed of pulley is to be 1060 revolutions per minute; horse-power to be transmitted, 100; coefficient of friction, 0.2: thickness of belt to be $\frac{24}{64}$ inches, and working strength 300 pounds per square inch. What should be its width?

401. A main driving belt is to encircle half of a 54-inch pulley. The speed of pulley is to be 350 revolutions per minute; horse-power transmitted, 520; coefficient of friction, 0.2. Thickness of belt, according to specifications, is to be $\frac{28}{64}$ inches, and working strength 450 pounds per square inch. What should be its width?

402. A plain belt without dressing encircling one-half of a pulley, when just on the point of slipping has a tension of 1 000 pounds on the taut side. More machinery being put into use, rosin is thrown on the belt. If the tension on the slack side remains the same as before, and the belt is just on the point of slipping, what horse-power will be transmitted, diameter of pulley being 10 feet, and revolutions per minute, 140?

403. A singie fixed pulley, 6 inches in diameter, turns on an axle 2 inches in diameter; coefficient of friction, 0.2. A weight of 500 pounds is lifted by means of this pulley. Find the force P that is required.

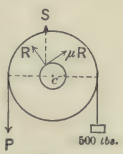


Fig. 65.

Friction causes the axle to creep, as it were, on its bearings. S moves a little off center, coming nearer to P. When the value of R can be found the friction will be determined by multiplying by μ. To find R:

$$S^2 = R^2 + \mu^2 R^2$$

as will be evident by plotting a parallelogram of force.

$$S = P + W$$

$$\therefore R = \frac{P + W}{\sqrt{1 + \mu^2}}$$

$$= \frac{P + 500}{1.02}$$

$$\mu R = 0.2 \times \frac{P + 500}{1.02}.$$

Now to find P:

Take moments about C, the center

$$-P \times 3 + 500 \times 3 + \mu R \times 1 = 0$$

$$P = 570 \text{ pounds.}$$

404. A shaft makes 50 revolutions per minute. The load on the bearing is 8 tons, the diameter of the bearing is 7 inches, and the average coefficient of friction is 0.05. At what rate is heat being generated?

$$S = P + W$$
$$= 8 \text{ tons}$$
$$R = \frac{P + W}{\sqrt{1 + \mu^2}}$$
$$= 15\ 980$$
$$\mu R = 799 \text{ pounds}$$
$$\text{Work of friction} = \text{force} \times \text{distance}$$
$$= 799 \times (\tfrac{7}{12} \times \tfrac{22}{7} \times 50)$$
$$= 73\ 240 \text{ foot-pounds per minute.}$$

405. A single fixed pulley, 2 feet in radius, turns on an axle 1 inch in radius; the weight of the pulley is 80 pounds. A weight of 500 pounds is lifted by means of this pulley. What force P is required? The coefficient of friction between axle and bearing is 0.1; the rope is flexible, and without weight, and P acts vertically.

406. Find the horse-power necessary to turn a shaft 9 inches in diameter making 75 revolutions per minute, if the total load on it is 12 tons and $\mu = .015$.

407. Let P and W be inclined to each other at an angle of 90°; radius of pulley is 6 inches; radius of axle $\frac{3}{4}$ inch; coefficient of friction, 0.2. Determine the relation of P and W in case of incipient motion.

408. A horizontal axle 10 inches in diameter has a vertical load upon it of 20 tons, and a horizontal pull of 4 tons. The coefficient of friction is 0.02. Find the heat generated per minute, and the horse-power wasted in friction, when making 50 revolutions per minute.

409. The shaft of a 1 000-kilowatt dynamo is 25 inches in diameter, makes 100 revolutions per minute, and carries a total load of 45 000 pounds. The coefficient of friction being 0.05, find the horse-power lost in heat that is generated by friction.

410. Find the horse-power absorbed in overcoming the friction of a foot-step bearing with flat end 4 inches in diameter, the total load being $1\frac{1}{2}$ tons, the number of revolutions 100 per minute, and the average coefficient of friction 0.07.

$$\underset{\text{of friction}}{\text{Work}} = W\mu \times \overset{\text{force}}{(\tfrac{2}{3}} \times \overset{\text{distance}}{\tfrac{2}{1\tfrac{1}{2}})} \times 2\pi \times 100.$$

The distance being obtained by considering a circumference as in problem 381, outside of which the work is the same as that inside. For a bearing with a flat end that circumference has a radius of two-thirds of R.

411. Calculate the horse-power absorbed by a foot-step bearing with flat end 8 inches in diameter when supporting a load of 4 000 pounds, and making 100 revolutions per minute, coefficient of friction 0.03.

412. A 150-horse-power turbine has an oak step 6 inches in diameter and with conical end tapering 45°. If the load on the step be 2 tons, and the

coefficient of friction between the wood and its metal seat be 0.3, find the horse-power thus absorbed at 65 revolutions per minute.

To resist the load of 2 tons would require a pressure of 2.83 tons by the 45° slope of the foot-step. The mean circumference would be as in preceding problems, distant two-thirds R from center.

413. The shaft of a vertical steam turbine has a conical foot-step bearing 3.5 inches in diameter, and length 3 inches. Total load on shaft, 1 500 pounds; speed 2 500 revolutions per minute; coefficient of friction, 0.07. Find the horse-power that tends to "burn out" the foot-step.

III. MOTION

414. A body moving with a velocity of 5 feet per second is acted on by a force which produces a constant acceleration of 3 feet per second. What is the velocity at the end of 20 seconds?

Velocity gained = acceleration per second × number of seconds.

$$v = a \times t$$
$$= 3 \times 20$$
$$= 60 \text{ feet per second.}$$

Final velocity $= 60 + 5$
$= 65$ feet per second.

415. The initial velocity of a stone is 12 feet per second; this velocity decreases uniformly at the rate of 2 feet per second. How far will the stone have traveled in 5 seconds?

416. Two trains A and B moving towards each other on parallel rails at the rate of 30 miles and 45 miles an hour, are 5 miles apart at a given instant. How far apart will they be at the end of 6 minutes from that instant, and at what distances are they from the first position of A?

417. Two trains, 130 and 110 feet long, pass each other in 4 seconds when going in opposite directions. The velocity of the longest train being double that of the other, find at what speed per hour each is going.

MOTION. Impact of Mallet and Polo Ball. Find Distance, Velocity, Time.

418. Two trains going in opposite directions pass each other in 3 seconds. One train is 142 feet long and the other 88 feet long. When going in the same direction one passes the other in 15 seconds. How fast is each train going?

419. The velocity of a train is known to have been increasing uniformly; at one o'clock it was 12 miles per hour; at 10 minutes past one it was 36 miles per hour. What was it at $7\frac{1}{2}$ minutes past one?

420. A train moving at the rate of 30 miles an hour is brought to rest in 2 minutes. The retardation is uniform. How far did it travel?

A railroad train is moving at 30 miles an hour. In each second, then, it moves 44 feet. Its velocity for each second during the time t may be represented as in Fig. 66 by lines of equal length, and the area of the rectangle, or vt, represents the distance passed over. This is an illustration of uniform motion.

When a railroad train starts from a station, and by uniform gain in speed attains a velocity of 30 miles an hour, the distance passed

Fig. 66.

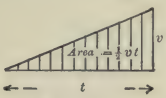


Fig. 67.

Fig. 68.

over may be graphically represented as in Fig. 67. The area would represent said distance.

421. Similarly, what condition of speed of the railroad train would Fig. 68 represent?

Note that the area of Fig. 68 is $v_0 t + \frac{1}{2}(v - v_0)\, t$, or $v_0 t + \frac{1}{2} a t^2$.

422. A stone skimming on ice passes a certain point with a velocity of 20 feet per second, then suf-

fers a retardation of one unit. Find the space passed over in the next 10 seconds, and the whole space traversed when the stone had come to rest.

423. On the New York Central and Hudson River Railroad test tracks near Schenectady, an electric locomotive hauled 9 Pullman cars at a running speed of 60 miles per hour. The average acceleration from start to full speed was 0.5 miles per hour per second. The retardation on applying air brakes was 0.88 feet per second per second. These results were obtained by carefully timing the train at measured stations. Total distance was 4 miles. What was the total time?

424. A train is running at the rate of 60 miles an hour when the steam is turned off; it then runs on a level track for $3\frac{1}{3}$ miles before stopping. If friction be the constant retarding force, find its value in pounds per ton. Also how far does the train run in 3 minutes from the instant steam is turned off?

425. A body acted on by a constant force begins to move from a state of rest. It is observed to move through 55 feet in a certain 2 seconds, and through 77 feet in the next 2 seconds. What distance did it describe in the first 6 seconds of its motion?

426. A steamer approaching a wharf with engines reversed so as to produce a uniform retardation is observed to make 500 feet during the first 30 seconds of the retarded motion and 200 feet during the next 30 seconds. In how many more seconds will the headway be completely stopped?

427. Two bodies are let fall from the same point at an interval of 2 seconds. Find the distance between them after the first has fallen for 6 seconds.

For 1st body, $s = \frac{1}{2} g t^2$
$= \frac{1}{2} \times 32 \times 6^2$
$= 576$ feet

For 2d body, $s = \frac{1}{2} g t^2$
$= \frac{1}{2} \times 32 \times 4^2$
$= 256$ feet

$\therefore$ distance apart $= 576 - 256$
$= 320$ feet.

428. A stone is projected vertically upwards with a velocity of 80 feet per second from the summit of a tower 96 feet high. In what time will it reach the ground, and with what velocity?

429. A hammer of 10 tons weight falling from a height of 4 feet drives a wooden pile and comes to rest in $\frac{1}{32}$ second. How far does it drive the pile? And assuming the force is uniform find it.

430. A stone is dropped into a well, and the sound of its striking is heard $2\frac{7}{12}$ seconds after it is dropped; the velocity of sound in air is 1 200 feet per second. What is the depth of the well?

Let $s =$ depth of well.

$\therefore$ time for sound to come up $= \frac{s}{1\ 200}$ seconds.

Time for stone to fall is found from formula

$$s = \tfrac{1}{2} g t^2$$

$$\therefore t^2 = \frac{2s}{g} = \frac{s}{16}, \quad \text{and} \quad t = \frac{\sqrt{s}}{4}.$$

Time for stone to fall + time for sound to come up $= 2\frac{7}{12}$.

$$\frac{s}{1\,200} + \frac{\sqrt{s}}{4} = \frac{31}{12}$$

$$s + 300\sqrt{s} = 3\,100$$

$$s \pm 150^2 + 300\sqrt{s} = 3\,100 \pm 150^2$$

$$\sqrt{s} = -310 \text{ an inadmissible value,}$$

or $$\sqrt{s} = +10$$

$$\therefore \quad s = 100 \text{ feet, depth of well.}$$

431. A stone is dropped from a tower of height a feet; another is projected upwards vertically from the foot of the tower; the two start at the same moment. What is the initial velocity of the second if they meet halfway up the tower?

432. A stone is dropped into a well, and the sound of the splash is heard 7.7 seconds afterwards. Find the distance to surface of the water, supposing the velocity of sound to be 1 120 feet per second.

433. A bucket is dropped into a well and in 4 seconds the sound of its striking the water is heard. How far did the bucket drop?

434. A balloon has been ascending vertically at a uniform rate for $4\frac{1}{2}$ seconds, and a test ball dropped from it reaches the ground in 7 seconds. Find the velocity of the balloon and the height from which the ball was dropped.

435. From a balloon that is ascending with velocity of 32 feet per second, a ball drops and reaches the ground in 17 seconds. How far up is the balloon?

436. A ball is let fall to the ground from a certain height, and at the same time another ball is thrown upwards with just sufficient velocity to carry it to the point from which the first one fell. When and where will they meet?

437. A cake of ice slides down a smooth chute that is set at an angle of 30° to the horizon. Through how many feet vertically will the cake of ice fall in the fourth second of its motion?

The acceleration for a body falling vertically is g, 32 feet per second per second. The acceleration component measured along a 30°-plane is $32 \times \sin 30°$, or 16 feet per second per second.

$$s = \tfrac{1}{2} at^2$$
$$= 72 \text{ feet, for 3 seconds}$$
$$= 128 \text{ feet for 4 seconds}$$

Therefore space along plane in the 4th second

$$= 56 \text{ feet}$$

438. A cable car "runs wild" down a smooth track of inclination 20° to the horizontal. How far does it go during the first 8 seconds after starting from rest?

439. A body is projected up a plan of 30° inclination with a velocity of 80 feet per second. How long before it will come to rest? How far will it go up the plane.

440. A body is sliding with velocity u down an inclined plane whose inclination to the horizon is 30°. Find the horizontal and vertical components of this velocity.

441. A stone was thrown with a velocity of 33 feet per second at right angles to a train that was going

30 miles an hour. It hit a passenger who was sitting on the opposite side of the car that was 9 feet wide. How far in front of him should be the hole in the window?

442. A deer running at the rate of 20 miles an hour keeps 200 yards distant from a sportsman. How many feet in front of the deer should aim be taken if the velocity of the bullet be 1 000 feet per second?

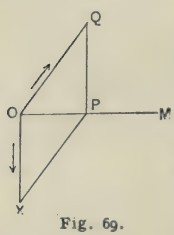


Fig. 69.

443. A boat is rowed at the rate of 5 miles an hour on a river that runs 4 miles an hour. In what direction must the boat be pointed to cross the river perpendicularly? With what velocity does it move?

Let OX be 4 units in length to represent the velocity of the stream.

Draw OM perpendicular to OX. The resultant velocity is to be in the direction OM.

With center X and radius of 5 units describe an arc cutting OM in P.

Join XP, and complete the parallelogram of velocities OXPQ.

OQ is the required direction.

The angle $QOP = \sin^{-1}\frac{4}{5}$.

Therefore the boat must not be rowed straight across, but up stream at an angle of 53° 10′.

To find the resultant velocity:

$$OP^2 = OQ^2 - QP^2$$
$$= 5^2 - 4^2$$
$$= 25 - 16$$
$$= 9$$
$$\therefore OP = 3$$

∴ the boat crosses the river at the rate of 3 miles an hour.

444. A river flows at the rate of 2 miles per hour. A boat is rowed in such a way that in still water its velocity would be 5 feet per second in a straight line. The river is 3 000 feet wide ; the boat starting from one shore, is headed 60° up-stream. Where will it strike the opposite shore ?

445. A bullet moving upwards with a velocity of 1 000 feet per second, hits a balloon rising with velocity 100 feet per second. Find the relative velocity.

446. A train at 45 miles an hour, passes a carriage moving 10 yards a second in the same direction along a parallel road. Find the relative velocity.

447. To a passenger in a train, raindrops seem to be falling at an angle of 30° to the vertical ; they are really falling vertically, with velocity 80 feet per second. What is the speed of the train ?

448. Two roads cross at right angles ; along one a man walks northward at 4 miles per hour, along the other a carriage goes at 8 miles per hour. What is the velocity of the man relative to the carriage ?

449. A steamer is going east with a velocity of 6 miles per hour ; the wind appears to blow from the north ; the steamer increases its velocity to 12 miles per hour, and the wind now appears to blow from the north-east. What is the true direction of the wind and its velocity ?

450. A ship is sailing north-east with a velocity of 10 miles per hour, and to a passenger on board the wind appears to blow from the north with a velocity of $10\sqrt{2}$ miles per hour. Find the true velocity of the wind.

451. A fly-wheel revolves 12 times a second. What is the angular velocity of a point on the rim taken about the center?

452. A broken casting flies along a concrete floor with initial velocity of 50 feet per second. The coefficient of friction being $\frac{1}{2}$ what will be its velocity after 3 seconds?

One of the axioms for problems in Motion is, that

$$P \text{ the force} : W \text{ the weight} = a : g.$$

The force producing motion : the total weight moved = the acceleration produced by the force : the acceleration that gravity would produce.

For the above example the force producing motion (or in this case retardation) is $W \times \frac{1}{2}$ and

$$W \times \tfrac{1}{2} : W = a : 32$$

$$a = 16 \text{ feet per second per second}$$

After 3 seconds the velocity would be

$$v = 50 - 16 \times 3$$

$$= 2 \text{ feet per second.}$$

453. A locomotive that weighs 100 tons is increasing its speed at the rate of 100 feet a minute. What is the effective force acting on it?

454. An ice boat that weighs 1 000 pounds is driven for 30 seconds from rest by a wind force of 100 pounds. Find the velocity acquired and the distance passed over.

455. A 5-pound curling iron is thrown along rough ice against a friction of one-fifth of its weight; it comes to rest after going a distance of 40 feet. What must have been its velocity at the beginning?

456. The table of a box-machine weighs 50 pounds and is pulled back to its starting position, a distance of 6 feet, by a falling weight of 20 pounds. What time, neglecting friction, will thus be used in return motion?

457. A body whose mass is 108 pounds is placed on a smooth horizontal plane, and under the action of a certain force describes from rest a distance of $11\frac{1}{9}$ feet in 5 seconds. What is the force acting?

458. Two bodies A and B, that weigh 50 pounds and 10 pounds, are connected by a string; B is placed on a smooth table, and A hangs over the edge. When A has fallen 10 feet, what is the accumulated work of the bodies jointly, and what of them severally?

459. A 500-volt electric motor imparts velocity to an 8-ton car so that at the end of 20 seconds it is moving on a level track at the rate of 10 miles an hour; the total efficiency of the motor and car is 60 per cent. What amperes are necessary?

460. Show that to give a velocity of 20 miles an hour to a train requires the same energy as to lift it vertically through a height of 13.4 feet.

461. What force must be exerted by an engine to move a train of weight 100 tons with 10 units of acceleration, if frictional resistances are 5 pounds per ton?

462. A train that weighs 60 tons has a velocity of 40 miles an hour at the time its power is shut off. If the resistance to motion is 10 pounds per ton, and no brakes are applied, how far will it have traveled when the velocity has reduced to 10 miles per hour?

The retardation a will be found to be 0.16 feet per second per second; the total loss in velocity is 44 feet per second. Then find the time, and lastly the space by observing that space = average velocity $\times$ time.

463. A locomotive running on a level track brings a train of weight 120 tons to a speed of 30 miles an hour in 2 minutes. The resistance to motion of the train being uniform and equal to 8 pounds per ton, what will be the required horse-power at the draw-bar and what the distance from the starting point when the speed of 30 miles an hour is attained?

464. A freight train of 100 tons weight is going at the rate of 30 miles an hour when the steam is shut off and the brakes applied to the locomotive. Supposing the only friction is that at the locomotive, the weight of which is 20 tons, what is the coefficient of friction if the train stops after going 2 miles?

$20 \times u : 100 = a$ (which can be found from the data given in the problem) : 32.

465. A train of 100 tons, excluding the engine, runs up a 1% grade with an acceleration of 1 foot per second. If the friction is 10 pounds per ton, find the pull on the drawbar between engine and train.

Total force = force for acceleration + force for lifting + force for friction.

466. A body is projected with a velocity of 20 feet per second down a plane whose inclination is 25°; the coefficient of friction being 0.4. Determine the space traversed in 2 seconds.

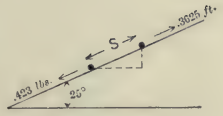


Fig. 70.

$$P : W = a : g$$

$$(.423 - .3625) \times W : W = a : g.$$

The space traversed,

$$s = Vt + \tfrac{1}{2} at^2.$$

467. A body slides down a rough inclined plane 100 feet long, the sine of whose angle of inclination is 0.6; the coefficient of friction is $\frac{1}{2}$. Find the velocity at the bottom. If projected up the plane with a velocity that just carries it to the top, find that velocity.

The forces acting down the plane

$$= W \times \sin a - W \times \cos a \times \tfrac{1}{2}.$$

468. An electric car at the top of a hill becomes uncontrollable and "runs wild" down a grade of 1 vertical to 20 horizontal a distance of $\frac{1}{4}$ mile. The resistance to friction being 20 pounds per ton and the total weight of car and passengers 50 tons, how fast will the car be going when it reaches the foot of the hill?

469. Two weights of 120 and 100 pounds are suspended by a fine thread passing over a fixed pulley without friction. What space will either of them pass over in the third second of their motion from rest?

Observe that the force producing motion is in this case 20 pounds, and the total weight moved is 220 pounds. Then $a = 2.92$ feet per second per second.

470. A man who is just strong enough to lift 150 pounds can lift a barrel of flour of 200 pounds weight when going down on an elevator. How fast is the velocity of elevator increasing per second?

471. A cord passing over a smooth pulley carries 10 pounds at one end and 54 pounds at the other. What will be the velocity of the weight 5 seconds from rest; and what will be the tension in the cord?

After computing the acceleration that the two weights would have, find the equivalent force, or tension, that would be required to cause said acceleration on the 10-pound weight, which is the one that is being moved. We have $a = 22$, and

$$P : 10 = 22 : 32$$

P, the tension = 6.9 pounds + 10.

472. Two strings pass over a smooth pulley; on one side both strings are attached to a weight of 5 pounds, on the other side one string is attached to a weight of 3 pounds, the other to one of 4 pounds. Find the tensions during motion.

473. Weights of 5 pounds and 11 are connected by a thread; the 11-pound weight is placed on a smooth horizontal table, while the other hangs over the edge. If both are then allowed to move under the action of gravity, what is the tension of the thread?

474. A 10-pound weight hangs over the edge of a table and pulls a 45-pound box along; the coefficient of friction between the table and the box is 0.5. Find the acceleration and the tension in the string.

475. An engine draws a three-ton cage up a coal-pit shaft at a speed uniformly increasing at the rate

of 5 feet per second in each second. What is the tension in the rope?

476. A balloon is moving upward with a speed which is increasing at the rate of 4 feet per second per second. Find how much the weight of a body of 10 pounds as tested by a spring balance on it, would differ from its weight under ordinary circumstances.

477. An elevator of 300 pounds weight is being lowered down a coal shaft with a downward acceleration of 5 feet per second per second. Find the tension in the rope.

478. An elevator, starting from rest, has a downward acceleration of $\frac{1}{2}\ g$ for 1 second, then moves uniformly for 2 seconds, then has an upward acceleration of $\frac{1}{3}\ g$ until it comes to rest. (*a*) How far does it descend? (*b*) A person whose weight is 140 pounds experiences what pressure from the elevator during each of the three periods of its motion?

479. A weight of 10 pounds rests 6 feet from the edge of a smooth horizontal table that is 3 feet high. A string 7 feet long passes over a smooth pulley at the edge of the table and connects with a 10-pound weight. If this second weight is allowed to fall in what time will it cause the first weight to reach the edge of the table?

480. A body is projected with a velocity of 50 feet per second in a direction inclined 40° upward from the horizontal. Determine the magnitude and direc-

tion of the velocity at the end of 2 seconds (g being taken equal to 32.15).

Let ACE be the path of the projectile. The vertical velocity which the body possessed when it started from A carried it to the summit C of the trajectory, where it had zero vertical velocity, and when it reached E it would possess its initial velocity, which would be $u \times \sin a$. (In Prob. 480 a is 40°.) The constant horizontal velocity would be $u \times \cos a$.

The vertical velocity acquired in falling from the highest point to the horizontal AE would be $g \times t$,

$$\therefore g \times t = u \times \sin a$$

and the time from A to highest point

$$t = \frac{u \times \sin a}{g}$$

and the total time of flight

$$= \frac{2u \times \sin a}{g}$$

The range AE

= the horizontal component of velocity × the time of flight

$$= u \times \cos a \times \frac{2u \sin a}{g}$$

$$= \frac{u^2 \sin 2a}{g}.$$

The above explanation and formulas will be of material assistance in solving the problems that follow. In all of these problems the resistances of the atmosphere are neglected.

481. A bullet is fired with a velocity of 1 000 feet per second. What must be the angle of inclination, in order that it may strike a point in the same horizontal plane, at a distance of 15 625 feet?

482. From the top of a tower a stone is thrown up at an angle of 30°, and with a velocity of 288 feet per second; the height of the tower is 160 feet. Find the time required for the stone to reach the ground, and the distance it will be from the tower.

483. From a train moving at 60 miles per hour a stone is dropped; the stone starts at a height of 8 feet above the ground. Through what horizontal distance will the stone go while falling?

484. A stone from a quarry blast has a velocity of 200 feet per second, in a direction inclined at an angle of 60° to the horizontal plane. To what height will it rise, and how far away will it strike the ground?

485. A bullet is fired with a velocity of which the horizontal and vertical components are 80 and 120 feet per second respectively. Find the range and greatest height.

486. The top of a fortification wall is 50 feet above the level of a city. From a man-of-war in the bay 300 feet below the top of the wall and distant horizontally 3 000 feet, a projectile is fired with velocity of 1 000 feet per second. The projectile just clears the wall. Where will it land inside the city?

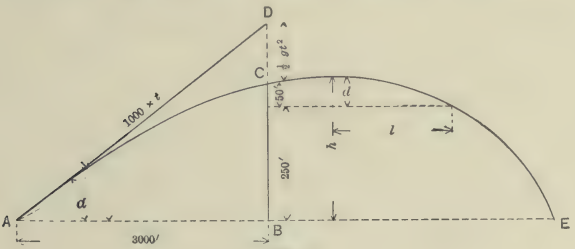


Fig. 71.

$$1\,000 \times t \times \sin a - \tfrac{1}{2} g t^2 = 300$$

$$1\,000 \times t \times \cos a = 3000$$

Eliminate t and solve for a ($a = 8° 28'$).

Find the greatest height h, then d being known will enable one to find the time for the projectile to fall that height or to pass horizontally over the distance l. To l add half the range and thus find the distance from man-of-war to where the projectile will land inside the city.

487. A ball is discharged with an initial velocity of 1 100 feet per second. How many miles is the greatest possible range?

488. A cannon ball is fired directly from a hill that is on the coast and 900 feet high: find the time which elapses before it strikes the sea.

489. A projectile is fired horizontally from the top of a hill 300 feet high to a ship at sea. Its initial velocity is 2 000 feet per second and its weight 500 pounds. What will be its range, and what will be the energy of the blow that it strikes?

490. What velocity must be given to a golf ball to enable it just to clear the top of a fence at 12 feet higher elevation and 100 yards distant, if the ball is struck upwards at an angle of 45°?

491. The explosive force of a shell is to be regulated by proper charging so that a required velocity can be attained. Find what velocity will be required for it just to clear a fortification wall the top of which is distant horizontally 1 mile and at elevation 300 feet above the gun. Angle of projection is 45°.

492. A rifle projects its shot horizontally with a velocity of 1 000 feet per second; the shot strikes the

ground at a distance of 1 000 yards. What is the height of the rifle above the ground?

493. What is the pressure exerted horizontally on the rails of an engine of 20 tons weight going round a level curve of 600 yards radius at 30 miles an hour?

$$\text{Centrifugal force} = \frac{W}{g}\,\frac{v^2}{r}.$$

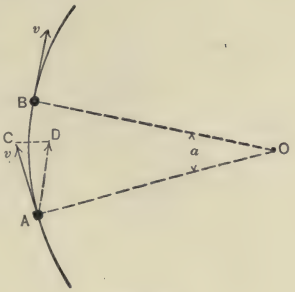


Fig. 72.

To derive the above formula:

Let A and B be two positions of the engine. At A the velocity, which is 30 miles an hour, would be in the direction of tangent v, and at B the same velocity would be in direction of tangent v. In going from A to B the direction of the velocity has changed, and the measure of this change is the centrifugal force.

Its value depends upon the *rate* of change of motion. To find the rate, from A draw AD to represent the velocity of position B. Then CD will represent the velocity of B relative to A, and its value will be $AB \times \frac{v}{r}$, as found from similar triangles ACD and ABO; and AB = velocity (on the curve) × time, so that CD, the velocity of B relative to A $= vt \times \frac{v}{r}$, and the rate of change $a = vt \times \frac{v}{r} \div t = \frac{v^2}{r}$. Thus knowing the rate of change or acceleration a the centrifugal force c can be found.

$$c : W = a : g$$

$$c = \frac{W}{g}\,\frac{v^2}{r}.$$

494. A train of 60 tons weight is rounding a curve of radius one mile, with a velocity of 20 miles an hour. What is the horizontal pressure on the rails?

495. A 24-ton engine is rounding a curve of 400 yards radius; the horizontal pressure on the rails is 4.84 tons. What is the velocity of the engine?

496. The rim of a pulley has a mean radius of 20 inches; its section is 6 inches broad and $\frac{1}{2}$ inch thick. It revolves at 200 revolutions per minute. What is the centrifugal force per inch length of rim?

497. The mass of the bob of a conical pendulum is 2 pounds, the length of the string is 3 feet, the angle of inclination to vertical is 45°. What is the tension?

The three forces acting on the bob are: its weight downward, the tension in the string, and the centrifugal force outward.

498. The mass of the bob is 20 pounds, the length of the string is 2 feet, the tension of the string is $500\pi^2$ pounds weight. How many revolutions per second is the pendulum making?

499. If a conical pendulum be 10 feet long, the half angle of the cone 30°, and the mass of the bob 12 pounds, find the tension of the thread and the time of one revolution.

500. A ball is hung by a string in a passenger car which is rounding a curve of 1 000 feet radius, with a velocity of 40 miles an hour. Find the inclination of the string to the vertical.

501. A ball is hanging from the roof of a railroad car. How much will it be deflected from the vertical when the train is rounding a curve of 300 yards radius at speed of 45 miles an hour?

502. Find the speed at which a simple Watt Governor runs when the arm makes an angle of 30° with the vertical. Length of arm from center of pin to center of ball, 18 inches. (Fig. 73.)

Fig. 73.

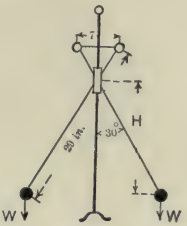


Fig. 74.

503. Find the speed of a cross-arm governor when the arms make an angle of 30° with the vertical. The length of the arms from center of pin to center of ball is 29 inches; the points of suspension are 7 inches apart. (Fig. 74.)

504. The rotating balls on a centrifugal governor make 160 revolutions per minute; the distance from the center of each ball to the center of the shaft is 4.5 inches. The balls are of cast iron and $2\frac{1}{2}$ inches in diameter. Find the centrifugal force of the governor.

505. Find the speed in revolutions per minute of a cross-arm governor when the arms make an angle of 30° with the vertical, the length of the arms from center of pin to center of ball being 24 inches, and the points of suspension being 6 inches apart.

506. Find the tension in each spoke of a six-spoked flywheel, 8 feet in diameter and weighing 1344 pounds when making 200 revolutions per minute assuming all its mass collected at its rim, and that by reason of cracks in the rim, the spokes have to bear the whole of the strain.

507. The flywheel, which burst in the Cambria Steel Company's mill Jan. 21, 1904 killing three men and seriously injuring nine more, is reported to have weighed 50 tons, and to have been about 20 feet in diameter. If rim weighed 35 tons and its weight was acting at a mean diameter of 19 feet what centrifugal force did each of the 8 spokes and its portion of the rim have to withstand when the wheel was "racing" at 150 revolutions per minute?

508. A man claims that he was injured by a horse-shoe that was thrown from the front wheel of a passing automobile. The rubber tire, he says, caught up the horse-shoe by a protruding nail and carried it around to the top of the wheel when it was thrown off with full force. Diameter of wheel was 30 inches, and speed of automobile was 20 miles an hour. What velocity would the rim of the wheel thus give to the horse-shoe?

509. The automobile of the above problem was turning the corner in a curve of 100 feet radius, the road being level. Therefore what two centrifugal forces were acting at the instant the 1-pound horse-shoe was at the top of the wheel? What were their values?

510. If the horse-shoe was thrown with the full velocity of the rim and horizontally from the top of it how far away would it land on level ground?

511. As the man was standing 6 feet from the track of the automobile how far from him must have been the wheel when the horse-shoe was thrown off, provided it was thrown tangentially to track?

512. A locomotive that weighs 35 tons runs at 40 miles an hour on a level grade round a curve of 3 300 feet radius (about 1° 44′). What centrifugal force is produced? What should be the elevation of the outer rail for a standard gage track of 4 feet $10\frac{3}{4}$ inches?

513. In the case of problem 512 the railroad contemplate putting on a 60-ton locomotive and running at maximum speed of 60 miles an hour. What lateral pressure will the spikes of the rails, if not changed, then have to withstand?

514. A stone weighing four ounces is whirled around the head 90 times a minute. If the sling is 3 feet 6 inches long what will be the pull in it?

515. Given l the length of a simple pendulum, $\pi\sqrt{\frac{l}{g}}$ the time of an oscillation: show how to find approximately the height of a mountain when a seconds pendulum, by being taken from sea level to its summit, loses n beats in 24 hours. If $n = 15$, what is the height of the mountain, the radius of the earth being 4 000 miles?

516. At sea-level a pendulum beats seconds. At the top of a mountain it beats 86 360 times in 24 hours. What is the height of the mountain?

517. A pendulum of length 156.556 inches oscillates in two seconds at London. What is the value of g?

518. An 800-pound shot is fired from an 81-ton gun, with a muzzle velocity of 1 400 per second: a steady resistance of 9 tons begins to act immediately after the explosion. How far will the gun move?

An impulsive force is a very large force that acts on a body for so short an interval of time that the body has practically no motion, but receives a change of momentum; and this change of momentum measures the Impulse or effect produced by the Impulsive Force.

In the above problem the impulsive force, or action on the shot to drive it forward, is equal to the reaction on the gun to drive it backward.

Action = reaction

Momentum before = momentum after

Momentum of gun backward = momentum of shot forward

$$\frac{W}{g} u = \frac{W'}{g} v',$$

and this simple formula, with a knowledge of the principles of work, will solve many problems that involve questions of momentum of two or more bodies.

For the above problem:

$$\tfrac{800}{2000} \times 1\,400 = 81 \times v$$

$v = \tfrac{560}{81}$ feet per second, velocity of gun at beginning of its motion.

s = average velocity $\times$ time.

To find the time: Motion has been retarded by a force of 9 tons and the weight thus retarded is 81 tons. Find a the rate of retardation; then since the velocity of $\frac{560}{81}$ equals $a \times t$, t can be found and lastly the space.

519. A 56-pound ball is projected with a velocity of 1 000 feet per second from an 8-ton gun. What is the maximum velocity of recoil of the gun?

520. A one-ounce bullet fired out of a 20-pound rifle pressed against a mass of 180 pounds, kicks the latter back with an initial velocity of 6 inches per second. Find the initial velocity of the bullet.

521. A shell bursts into two pieces that weigh 12 pounds and 20. The former continues on with a velocity of 700 feet per second, and the latter with a velocity of 380 feet per second. What was the velocity of the shell when the explosion occurred?

522. A man weighing 160 pounds jumps with a velocity of $16\frac{1}{4}$ feet per second into a boat weighing 100 pounds. With what velocity will boat move away?

523. A freight train weighing 200 tons, and traveling 20 miles per hour, runs into a passenger-train of 50 tons standing on the same track. Find the velocity at which the broken cars of the passenger train will be forced along the track, supposing $e = \frac{1}{5}$.

Momentum before = momentum after.

Now with this formula combine a second law, namely:

The differences in velocities before × some constant = the difference in velocities after.

$$1. \quad \frac{W}{g}u + \frac{W'}{g}u' = \frac{W}{g}v + \frac{W'}{g}v'$$

$$2. \quad (u' - u)e = v - v'$$

(u and u' are velocities before impact; v and v' after impact.)

Solving these equations, and

$$v = \frac{Wu + W'u' - eW'(u - u')}{W + W'}$$

$$v' = \frac{Wu + W'u' + We(u - u')}{W + W'}.$$

524. A freight car of 20 tons weight is switched on to a siding with velocity of 15 miles an hour, and collides with another car of 10 tons weight that is moving in the same direction at 5 miles an hour. If the coefficient of impact is $\frac{1}{5}$, find the velocities of the cars after they collide.

525. A ball of mass 4 pounds and velocity 4 feet per second meets directly a ball of mass 5 pounds with opposite velocity of 2 feet per second; $e = \frac{1}{2}$. Find the velocities after impact.

526. An 8-pound bowling ball going 12 feet a second overtakes and strikes directly a 10-pound ball going 6 feet a second. Find their velocities after striking when the coefficient of impact e is $\frac{3}{4}$.

527. A body weighing 10 pounds, and moving at the rate of 15 feet a second, strikes another body B weighing 20 pounds, and moving at the rate of 10 feet a second, in the direction at right angles to that of A's motion. The bodies are to be treated as points, and the impact is supposed to take place in the direction of A's motion. Find the velocities and directions of the motions of the bodies after impact, the restitution being perfect (coefficient of elasticity = 1).

Plot a figure to show the conditions. The velocity of each in a direction perpendicular to a line through their centers is unchanged by the impact.

Note that u of formula for problem 523 becomes $u \times \cos 45°$; u' becomes $u' \times \cos 90°$; v, $v \cos 180°$; and v', $v' \cos 45°$.

IV. REVIEW

Many of the review problems that follow have been prepared from actual engineering conditions. They are classified somewhat according to their subject matter, but are not given in graded order, nor with solutions, as they are intended especially for students who have already pursued a course in Mechanics, or for engineers in practice who may wish to "brush up a bit."

528. One of the largest chimneys in America is that of the Clark Thread Co. at Newark, N.J. Its height is 335 feet, interior diameter 11 feet, outside diameter at base $28\frac{1}{2}$ feet, at top 14 feet. Find the work done in raising the material from the ground to its place in the chimney.

The volume may be determined by considering the whole cone and subtracting the top and the core. The average height to raise the material is found to be 119.07 feet; the average weight of material is 130 pounds per cubic foot.

529. By tests at the U.S. Naval Academy a concrete pile of conical shape 19 feet long and tapering from 6 inches at the point to 20 inches at the head, was driven $\frac{7}{8}$ inches by 2 blows from a 2 100-pound hammer falling 20 feet. The same hammer and fall by two blows drove a wooden pile also 19 feet long, but $9\frac{1}{2}$ inches at the point and 11 inches at the head, a distance of $5\frac{5}{16}$ inches. The report says: "This shows the comparative bearing power."

What were the resistances (*a*) of the concrete pile (*b*) of the wooden?

Fig. 75. Half-way Home.

530. A 60-inch McCormick water turbine at the Talbot Mills in North Billerica, Mass. was tested by the author and engineering students in December, 1905.

The quantity of water entering the turbine was, by measurements with a Price current meter, found to be 7 946 cubic feet per minute, with speed gate open 40 per cent; net fall given by difference in reading of gauges in forebay and raceway, 10.7 feet. How many horse-power was the water delivering to the turbine, that is, what was the "input?"

531. The power available for manufacturing purposes was measured by a friction brake. (See Fig. 76.) Length of arm was 12.00 feet, revolutions of pulley 100 per minute, net reading on platform scales for the above test was 400 pounds. What

horse-power was developed, that is, what was "the output?" "The in-put" by problem 530 being 161 horse-power, what was at that time the efficiency of the turbine with its set of bevel gears and 50 feet of horizontal shafting?

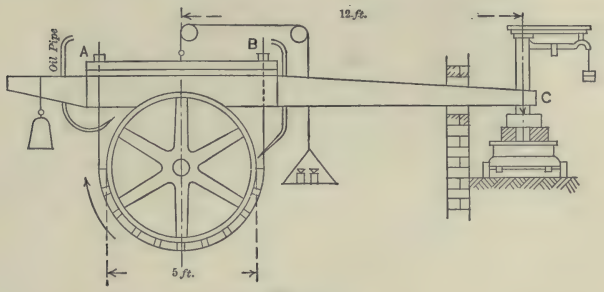


Fig. 76.

532. The strap underneath the pulley in Fig. 76 is tightened by the nuts A and B on top of the lever. When the brake is in use one nut can be tightened by a pull of 50 pounds on a 3-foot wrench, the other by 25 pounds.

Which nut should tighten harder and what will be the approximate tension in that end of the strap, the threads being 6 to the inch with mean diameter of $1\frac{1}{2}$ inches, and the hexagonal nut having a mean outside diameter of $2\frac{3}{4}$ inches? Use 0.25 as the coefficient of friction between the nut and its washer, and 0.15 for the threads.

533. For a bridge like Fig. 77 with lower chord a parabola 200 feet span and 30 feet rise what would be the stresses at the abutments and middle when the vertical reaction at each end is 600 000 pounds?

Fig. 77. The Driving Park Bridge at Rochester, N. Y.

534. The platform of a suspension foot-bridge 100 feet span, 10 feet width, supports a load, including its own weight, of 150 pounds per square foot. The two suspension cables have a dip of 20 feet. Find the force acting on each cable close to the tower, and in the middle, assuming the cable to hang in a parabolic curve.

535. Find analytically the stress in the cable at a horizontal distance of 30 feet from the center.

536. The weight of a fly-wheel is 8 000 pounds and the diameter, 20 feet; diameter of axle, 14 inches; coefficient of friction, 0.2. If the wheel is disconnected from the engine when making 27 revolutions per minute, find how many revolutions it will make before it stops.

537. Experiment shows that a weight can lift only three-quarters of its own weight by means of a rope over a single pulley, this being on account of the stiffness of the rope and the friction of the axis. Hence show that the mechanical advantage of four such pulleys arranged in two blocks is about 2.05.

Fig. 78. Concrete dam falling into place, Niagara, N. Y.

Height of trestle is 20 feet, of dam 50 feet, cross section of dam 7 feet, 4 inches square, and base of trestle 1 foot wider on the water side. Tower was tipped by three heavy jacks until it fell, as shown in cut. (See Eng. Record of Nov. 18, 1905.)

538. How many feet out of plumb would the top of tower move before starting to fall?

539. What foot-pounds of energy did the falling tower possess at the instant it passed the level of the base?

540. A bolt, $1\frac{1}{4}$ inches mean diameter, 9 threads to an inch, head 2 inches outside diameter, is tightened up by a wrench 12 inches long, and pull on end of 50 pounds. μ for threads is 0.2, for nut 0.3. Find the stress in the bolt.

541. In a certain piece of street railway construction I observed that the bolts in the fish plates were being tightened unusually hard, and the workmen told me that such bolts often broke. By test we found that a pull of 170 pounds was being used at a distance of 3.75 feet from center of nut. There were 9 threads to the inch, mean diameter 0.86 inches, average outside diameter of nut 1.4 inches, diameter of inside bearing 0.89 inches. The coefficient of friction for the threads was about 0.2 and for the nut 0.3.

Find stress in bolt, and then the stress per square inch at root of thread which was 0.80 inches in diameter.

542. The mean diameter of the threads of a $\frac{1}{2}$ inch bolt is 0.45 inches, the slope of the thread .07, the mean circumference of nut has a radius of 0.38 inches and the coefficient of friction 0.16. Find the tension in the bolt when tightened up by a force of 20 pounds on the end of a wrench 12 inches long.

543. The floating cantilever crane shown in the illustration is hoisting a 90-ton turret off the U. S. Monitor "Florida." Distance from middle of crane to the legs is 45 feet; from legs to turret 15 feet. Vertical height of legs 60 feet, distance apart at top

10 feet, at bottom 50 feet. Find the stresses for this case in the legs, and in the inclined members at center considering a joint at that point.

544. The load of 90 tons is supported by two large steel blocks. Each block has four 4-foot steel sheaves thus using 8 strands of $1\frac{1}{8}$ inch cast-steel rope. The

Fig. 79.

speed of vertical travel is 50 feet per minute. (*a*) What will be the velocity of travel of the hoisting strand? (*b*) What will be the stress per square inch in each rope of each block?

545. Launching data for nine of our recent warships was given in papers read before the Society of Naval Architects and Marine

Engineers at New York, November, 1904, and published in abstract by Engineering News, Dec. 22, 1904.

For the "California," built by the Union Iron Works of San Francisco, and launched April 28, 1904:

Total moving weight, ship, cradle, etc. . . . 6 062 tons
Mean slope of upper end of ways . . . . . 1 in 27.6

Ship started "very slowly." It moved the first foot in 11 seconds.

From the above the initial coefficient of friction has been computed as .036. Check this result.

546. Total travel of the ship, in preceding problem, was 786 feet; total time, 1 minute 16 seconds; Travel to point of maximum velocity, 320 feet; time to maximum velocity, 41.5 seconds; amount of maximum velocity, 22.1 feet per second. The weight of ship exclusive of cradle was 5 980 tons. Means for checking were rope stops (72 were broken, each of strength 30 tons), anchors, and mud banks. Find the average resistance that the means of checking must have afforded.

547. Consult the reference of problem 545 (Eng. News, Dec. 22, 1904), and copy Fig. 6 in note-book. Explain why the velocity and distance curves start horizontally from the origin. In the acceleration curve what was the cause of the rise between 15 and 30 seconds of elapsed time?

548. From curve of problem 547 what was the velocity at time of 41.5 seconds? The acceleration? What was the acceleration at 55 seconds? From that data compute the velocity for 55 seconds.

549. Coal from a barge (Figs. 80 to 82) is hoisted to a steeple tower where it is run into a car and by the action of gravity alone the car goes down a grade 294 feet long in 24 seconds. It strikes a cross-bar, or "stopper" which is pushed back a distance of 30 feet while the car empties and for an instant comes to rest. The weight of the car is 2 000 pounds and of the coal 4 000. If the car empties uniformly during the 30 feet, what is the average force of resistance that the cross-bar exerts?

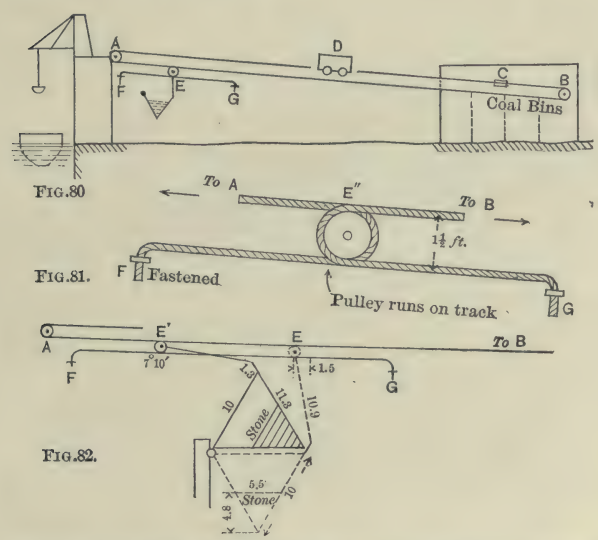


FIG. 80.

FIG. 81.

FIG. 82.

550. The method of stopping the car of problem 549 may be understood by referring to Figs. 80 to 82. The car, going down the grade, picks up the cross-bar C, which is clamped to the wire cable AB. As the cross-bar is pushed along the cable moves, and the

pulley E, around which the cable passes, as shown in the enlarged sketch E″, goes from E, its initial position, to E′, its final position. The travel of this pulley raises a triangular frame, that is partly filled with broken stone, from the position shown dotted to that shown by full lines in Fig. 82. The mass of stone is 5.5 feet × 4.8, as shown, and 1½ feet thick. The space is one-third voids; weight of stone, 150 pounds per cubic foot; weight of wooden frame, 1 000 pounds.

Show that the force exerted parallel to the cable is, for the initial position with pulley at E, about 220 pounds; for the final position E′ about 1925 pounds.

551. When the cross-bar and wire cable of the preceding problems move through a distance of 30 feet the travelling pulley E goes from E to E′ as described. The diameter of pulley being 1½ feet what will be the distance E E′? What would be the distance if pulley were 2 feet in diameter?

552. One-fourth of the energy possessed by the mass of stone when in the final position E′, Fig. 82, is lost in friction, and three-fourths of it is utilized to "kick" automatically the empty car up the incline from C back to its starting point A. If this energy is expended on the car in a return distance of 30 feet what will be the maximum velocity of the car as it starts back?

553. When the bridge of Fig. 83 carries a crowd of people making a load of 150 pounds per square foot what will be the reactions for the middle truss and the stresses in the inclined and horizontal members at the abutments?

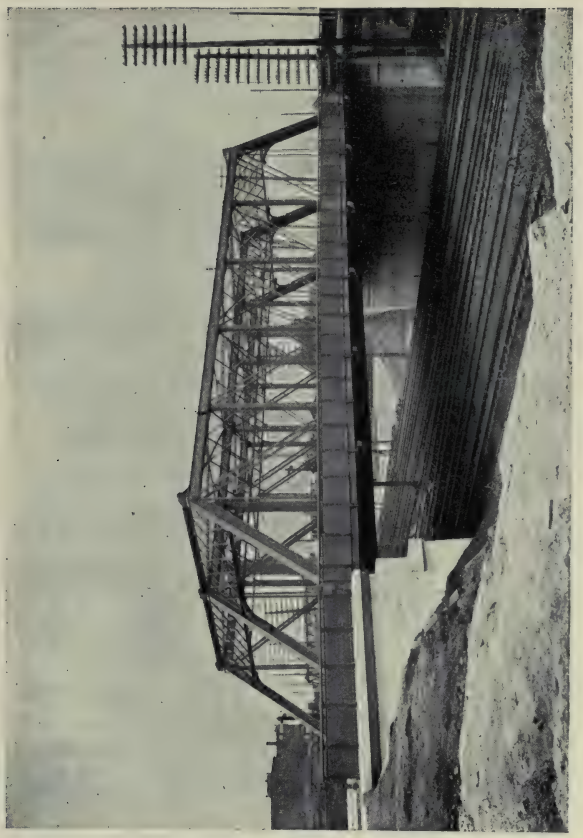


Fig. 83.

A modern highway bridge over the main tracks of the Reading railroad at Seventeenth and Indiana Streets, Philadelphia. Three Pratt trusses, with 24-feet clear roadways and two 10-foot sidewalks outside. Concrete abutments. The middle truss is 135 feet $0\frac{3}{8}$ inches from center to center of end pins, and 22 feet from center to center of chords. Equal panels.

554. A bridge of the type shown in Fig. 83 is sued for a double-track railroad. Length of bridge is 150 feet, there are 6 equal panels, and height of trusses is 25 feet. With loading of Fig. 85 and the second driver of forward locomotive placed at the first panel point from abutment, what will be the stresses in the inclined member and the first panel of lower chord of the truss which is carrying two-thirds of the loadings?

555. Span 11 of the Benwood Bridge on the Baltimore and Ohio Railroad is 347 feet long. It was reconstructed in 1904 and designed to carry the heavy loading shown in Fig. 85. What would be the reactions for a train half-way across the bridge?

556. With the forward truck just going off the bridge, what will be the reactions?

557. The locomotive of the Empire State Express has four drivers and a total weight of 124 000 pounds; the weight on the drivers is 84 000 pounds; the coefficient of friction between wheels and rails is 0.18. Find the total weight of itself and train which it can draw up a grade of 1 in 100, if the resistance to motion is 12 pounds per ton.

558. From the following data given by a General Superintendent, determine what revolutions per minute the locomotive drivers were making:

"The driver wheels are 42 inches in diameter, each pair being geared to a 200 HP. 625 volt-motor, with ratio of gearing 81 to 19, providing for a total tractive effort at full working load on 8 motors of 70 000 lbs. and at starting of 80 000 lbs., assuming 25% tractive coefficient, giving a nominal rating of 1 600 HP. The free running speed of these locomotives is about 20 miles per hour."

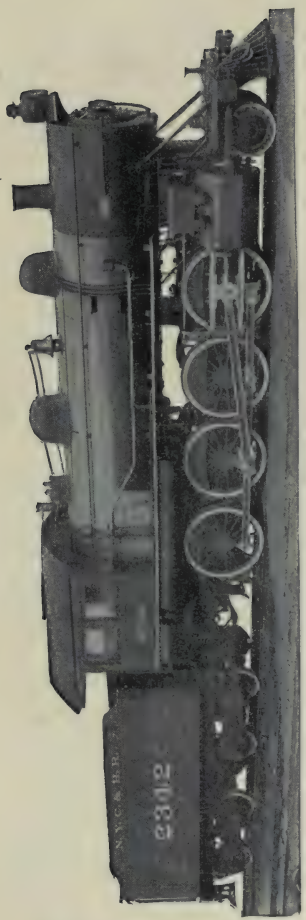


Fig. 84.

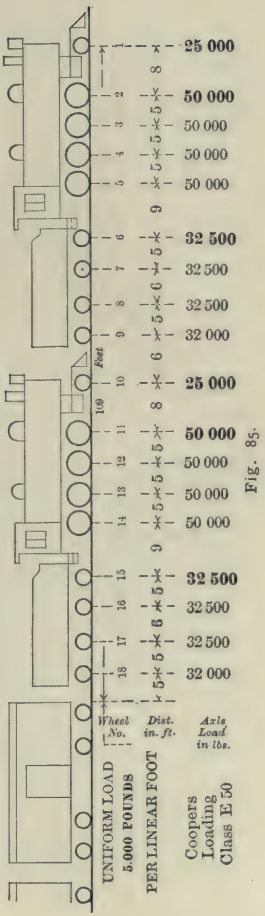


Fig. 85.

Fig. 84 shows a consolidated locomotive typical of modern design and development; Fig. 85, two such locomotives and their train load as used in computations of modern railroad bridges.

559. Also from the foregoing data determine what revolutions the motors were making. What amperes were supplied to the 8 motors?

560. An enormous freight locomotive — a Mallet duplex compound — designed as a "mountain helper" was put in service on the Baltimore and Ohio Railroad in January, 1905. This locomotive has drawn 36 steel cars weighing 702 tons, and 1 668 tons of lading, up a 1% grade, and with an average speed of $10\frac{1}{2}$ miles an hour; weight of locomotive with tender and an average amount of coal and water is 225 tons. What horse-power without friction was developed for hauling the above total load?

561. What per cent of the work done in the preceding problem would be paying work?

562. The draw-bar pull of this Mallet Compound has been found to be 74 000 pounds. When running with conditions according to problem 560 what frictional resistances would exist?

563. At the testing-plant of the Pennsylvania Railroad at the St. Louis Exhibition in 1904 a freight locomotive of type two-cylinder cross-compound consolidation (2·8·0), size 23 &35 × 32, made by the American Locomotive Company for the Michigan Central Railroad, gave the following data: Driving wheels 5 feet 3 inches in diameter, total weight 189 000 pounds, on drivers 164 500; maximum tractive effort, sand being used and locomotive acting as a compound, was 31 838 pounds.

According to the above data what would be the coefficient of friction between drivers and rails?

564. With speed of 15.01 miles per hour, 80.18 revolutions per minute, piston speed of 428 feet per minute, the indicated horse-power was 734.9; dynamometer horse-power 675.7. Find the draw-bar pull and the per cent of indicated horse-power that was lost in friction.

565. A car is supported on four 36-inch wheels with 4-inch axles and coefficient of friction 0.05. What traction will be required to move the car on a level track with a total weight of 20 ton on the axles? What energy will be lost in friction per minute with the car moving 30 miles an hour?

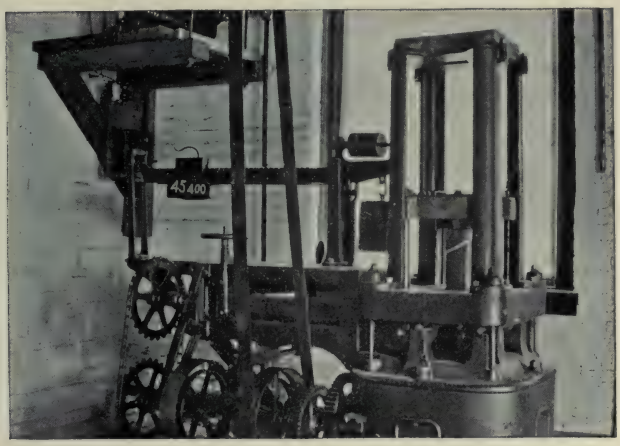


Fig. 86.

A block of maple wood 8 inches long and 2 × 3 inches in cross-section is being tested in a 60 000-pound Olsen testing machine.

The load on the test piece at the time of failure is 45 40c pounds. The plane of fracture is shown by the white line on the test-peice, Fig. 86. This plane makes an angle of 23° with the horizontal.

566. What would be the pressure in the direction of the plane at the time of breakage? Find the number of square inches thus resisting this pressure and then the stress per square inch — which is known in applied mechanics as the Shear.

567. A horse is pulling a 300-pound cake of ice up a plank run which makes an angle of 40° with the horizontal. There are two single pulleys which have efficiencies of 80% each. Coefficient of friction on plank run 0.05. What pull must horse exert?

568. A ring of weight W is free to slide on a smooth circular wire that stands in a vertical plane. A string attached to the ring passes over a smooth pin at the highest point of the circle and sustains a weight P. Determine the position of equilibrium.

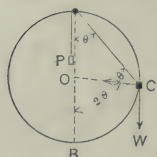


Fig. 87.

569. CD is a vertical wall. A is a point of support 12 feet from the wall. ED is a uniform bar 32 feet long resting on A and against the wall CD. All the surfaces are smooth. Find the position of equilibrium of the bar.

570. Fig. 88 shows a Carson-Lidgerwood cableway in use at Hartford, Conn., lowering a 12-foot length of 36-inch cast-iron pipe. The pipe is part of an intercepting sewer that passes under a river by means of an inverted siphon. Weight of pipe is 3 tons, span of cableway 300 feet, and the design of cableway is such that,

Fig. 88.

as customary, the sag by a full load in the middle is allowed to be $\frac{1}{20}$ of the span.

The practical method used by manufacturers of trench machinery for computing the total stress in cableways of this sort is to consider the condition of maximum loading, namely with the load in the middle of the span, and for that condition to divide half the span by double the sag $\left(\frac{\frac{1}{2}\text{ span}}{2\text{ sag}}\right)$. This gives a factor which mul-

tiplied by the total load (in this case 3 tons + weight of cable) gives the required stress in the main cable

How does the result obtained by the above practical method check with result obtained by the method of parallelogram of forces, considering the total load of 3 tons + weight of cable as acting at middle of span?

571. Each tower for the above cableway is 30 feet high, and consists of a vertical timber frame, or bent, that is formed by two 8 × 10 inch legs spread 9 feet apart at the bottom and 2 feet at the top. The main cable passes over an iron saddle at the top of the tower and is fastened to a "dead-man" anchorage that is buried 60 feet from the foot of the tower. The timber frame is kept vertical by steel guys made fast at top of tower and to the same "dead man."

What horizontal stress do these guys have to provide for when the main cable is free to slip on the saddle? What vertical load do they add to the tower?

572. The pull on the hoisting ropes causes an additional stress that would be, for this case, equivalent to a vertical load of about 3 tons on one tower. Add the three vertical stresses due to main cable, steel guys, and hoisting ropes and then find the stress in each leg of the tower.

573. A 12-inch Pelton water-motor of 3 horse-power is tested by a friction brake that encircles three fourths of a 4-inch pulley on the motor and has a lever arm that extends 22 inches from center of pulley to scales. The scales read 5 pounds when motor is making 1 150 revolutions per minute. What horse-power is being developed?

574. A 6-inch water-pipe that is 600 feet long is delivering 750 gallons of water per minute; the water is shut off by uniformly closing a 6-inch valve in 3 seconds of time. How much will the static pressure near the valve be increased?

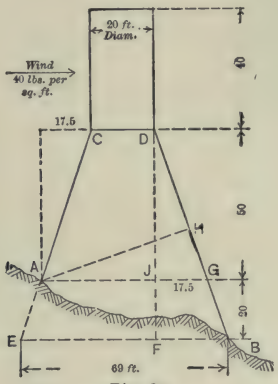


Fig. 89.

575. A water-works tank is on a trestle which stands on uneven ground as shown in diagram. The tank weighs 30 000 pounds. A strong wind gives a pressure of 40 pounds per square foot. Find the stress in the plane of the legs DB (*a*) when the tank is empty; (*b*) when the tank is full. (*c*) Find how much water will prevent the tank from overturning.

The wind acting on the curved surface of the tank causes a pressure that may be taken as 0.6 of that on a vertical section through the middle of the tank.

576. Name two advantages of the hemispherical (or similar) bottom over a flat bottom as represented in Fig. 89. A tank 20 feet high on the sides and 20 feet in diameter with hemispherical bottom will hold how many gallons of water? What will be the wind pressure if taken as in preceding problem?

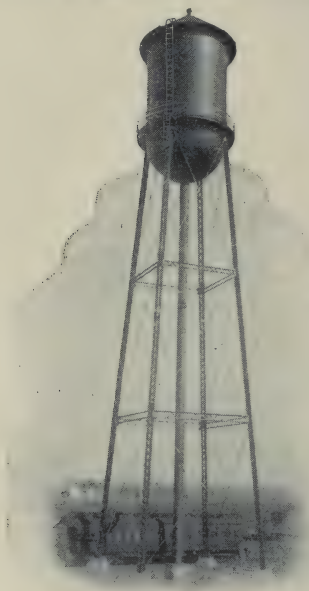


Fig. 90. A modern form of water-tank. (Erected at St. Elmo, Ill., by the Chicago Bridge and Iron Works.)

577. A clause in proposed specifications for water valves requires that "a valve shall stand without injury a pull of 175 pounds on a wrench that is in length $1\frac{1}{2}$ times the radius of the wheel." In considering these specifications an engineer and inspector asks if this test unduly strains a valve? The following analysis can be made relative to an 8-inch outside screw-and-yoke valve: Diameter of hand wheel is 14 inches, diameter of spindle, $1\frac{3}{8}$ inches, 4 threads to the inch (mean bearing diameter $1\frac{1}{4}$ inches). Valve seats taper from 4 inches to 2 inches in diameter of valve — 8 inches. Bearing of hand wheel has a mean diameter of 2 inches. The coefficient of friction for the bearing of hand wheel, the threads, and the face of valve against its seat, may be taken as 0.15.

Find the stress in the spindle caused by the pull of 175 pounds as indicated above.

To find this stress it is only necessary to consider conditions that affect the friction of the hand wheel. The resistances at the valve have no affect on the stress in the spindle which in any case is subjected to that part of the stress that is transmitted by the hand wheel. To compute this stress consider one revolu tion of the hand wheels.

$$\underset{\text{of pull}}{\text{Work}} = \underset{\text{of lifting}}{\text{Work}} + \underset{\text{on threads}}{\text{Work}} + \underset{\text{on bearing}}{\text{Work}}$$

Substitute and find the unknown term W, the stress.

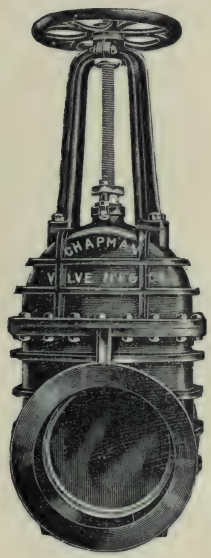


Fig. 91.

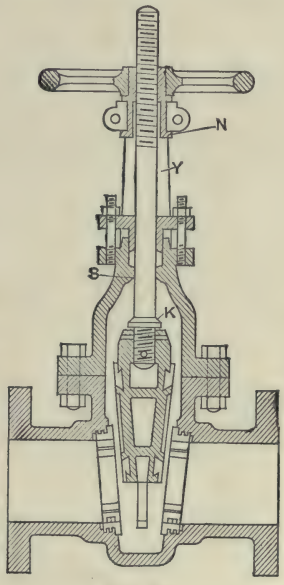
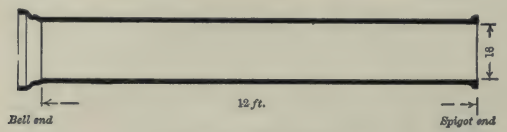


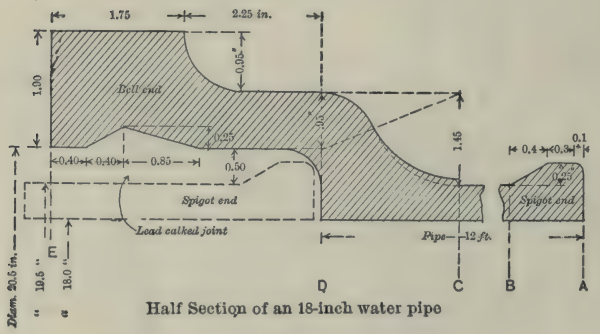
Fig. 92.

578. Find the pressure against the seat of the valve (no water pressure being considered) when a pull of 175 pounds is applied as in problem 577.

579. When a water pressure of 100 pounds per square inch acts on one side of the 8 inch valve in problem 578 and the test of 175 pounds pull is applied what normal pressure exists on each side of the valve?



A length of water pipe



Half Section of an 18-inch water pipe

Fig. 93.

Dimensions and weights for cast-iron water pipe are given in the "Standard Specifications for Cast-iron Pipe and Special Castings," issued Sept. 10, 1902, by the New England Water Works Association.

580. The dimensions for an 18-inch pipe of class D designed for a hydrostatic pressure of 300 pounds per square inch are represented in Fig. 93. Find the weight of portion E D and then the total weight of the whole length of pipe.

581. A roof has triangular trusses 12 feet apart. Weight of roof covering and snow equals 30 pounds per square foot, and the floor gives a load equivalent to 20 000 pounds concentrated at the foot of a vertical rod at the center of the truss; length of truss is 40 feet, height 10 feet. Find the stresses in rafters and tie-rod.

582. A triangular jib-crane ABC carries at A 60 000 pounds, the line of action being parallel to BC which is vertical. AB = 10 feet, BC 8 feet, AC 11 feet. Find the amount and kind of stresses acting in AB and AC.

583. A derrick with mast 40 feet long and boom 55 feet long, set at 60° from the horizontal, is lowering into water a wrought-iron pipe 12 feet long, 60 inches internal diameter, 66 inches external diameter. Density of wrought-iron is 7.8. Find the stresses in boom and tackle when the pipe is in air, and also when it is in water.

584. Is the retaining wall shown in Fig. 94 safe against overturning by the earth pressure acting as represented? Does the resultant pressure between the weight of the masonry, taken at 170 pounds per

cubic foot, and the earth pressure cut the base "within the middle third?"

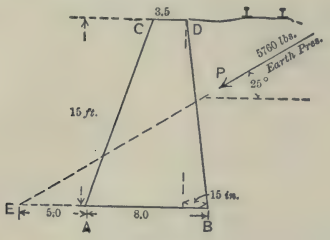


Fig. 94.

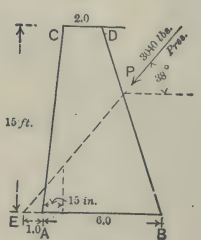


Fig. 95.

585. Fig. 95 shows a retaining wall of masonry as built at Northfield, Vt. As in the preceding problem, is the wall safe against overturning? Where does the resultant cut the base?

586. The waste gate of the canal for the Nashua Manufacturing Company at Nashua, N.H., is about $7\frac{1}{2}$ feet high and $4\frac{1}{2}$ feet wide. When this gate is closed there is usually a head of 10 feet of water on its center. The coefficient of friction of this wooden gate against an ordinary metal seat is taken as 0.40 and the weight of the gate is 1 000 pounds. What force in tons will be required to lift it?

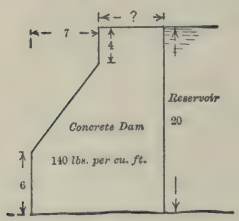


Fig. 96.

587. How wide on top should be the dam shown in Fig. 96 to withstand the reservoir pressure with factor of safety of 8?

588. Two Indians wanted to divide a birch log, that was 30 feet long and tapering from 8 inches in diameter to 12, so that each would have one-half. A school teacher told them to balance it and saw it open at that point. At what point should it be cut?

589. A 20 pound shot is fired from a 2 000-pound gun of length 10 feet; the muzzle velocity of the shot being 1 200 feet per second how far will the gun recoil up an incline rising 1 vertically to 15 on the slope? How long will it take the shot to travel through the gun?

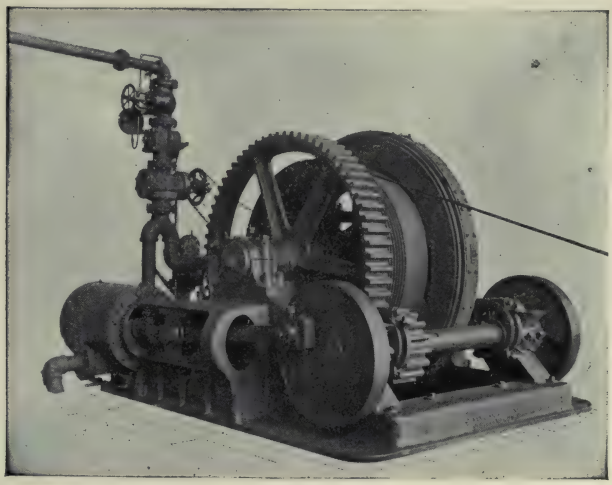


Fig. 97.

590. The engine and geared drum shown in the illustration are used for hoisting ore to the top of a blast furnace. The engine cyl-

inder is 12 inches in diameter, makes a 15-inch stroke, and 300 revolutions per minute. The mean effective pressure of the steam being 100 pounds per square inch, what horse-power is developed? The ratio of gears is 5.6. to 1; the diameter of drum, $4\frac{1}{2}$ feet. The efficiency of engine, geared drum and rest of mechanism is about 85 per cent.

Therefore, under the above conditions, what force will the engine give to the cable for drawing the loaded "skip-car" up the incline to the top of the blast-furnace?

591. The drum of a hoisting engine is 4 feet in diameter. The angle between the engine crank and connecting rod is 60°. Length of crank 1 foot, connecting rod 5 feet. Steam pressure on the piston 100 000 pounds which just balances a load W that is being hoisted. Determine the load W, the compression in the connecting rod and the side pressure against the cross-head guide.

592. Sixteen horse-power is to be transmitted by a belt which embraces $\frac{2}{9}$ of the circumference of a 20-inch pulley that makes 120 revolutions per minute; coefficient of friction is 0.35. Find (*a*) the tension in the two sides of the belt when slipping is just prevented and (*b*) the width of belt required, thickness being $\frac{3}{8}$ inches, and working stress 300 pounds per square inch of section.

593. Find the width of a belt necessary to transmit 10 horse-power to a pulley 12 inches in diameter, so that the greatest tension may not exceed 40 pounds

per inch of width when the pulley makes 1 500 revolutions per minute, and the coefficient of friction is 0.25.

594. A test was made, Aug. 15, 1905, of the new steam plant of the Wolff Milling Company at New Haven, Mo. The engine had a high-pressure cylinder 12 inches in diameter; low pressure, 24 inches; length of stroke, 36 inches. The revolutions were 77.1 per minute; the indicated horse-power of high-pressure cylinder, 85.74, low pressure, 66.19. What were the mean effective pressures in the two cylinders?

595. The engine and boiler test of problem 594 was continued 10 hours. During that time 177.72 barrels of flour (each weighing 196 pounds) had been made, and 3 685 pounds of coal had been burned in the boilers; cost of coal per ton of 2 000 pounds was $2.90. Find the cost of coal for each barrel of flour made, and the pounds of coal burned per hour per indicated horse-power.

596. A Columbus Gas Engine tested as shown by Fig. 98, Oct. 21, 1905, gave the following data: Revolutions during 15 minutes 3 688, explosions 1 516, net load on brake arm 30 pounds, length of arm 5 feet 3.024 inches. Four indicator cards taken during the test gave average areas 0.69 square inches, length 3.00 inches. Stiffness of spring that was used 300.

The diameter of engine cylinder is 7.76 inches and the length of stroke 11 inches. Find the efficiency.

In computing the horse-power of a gas engine by indicator cards the number of explosions corresponds to the number of revolutions for an ordinary engine.

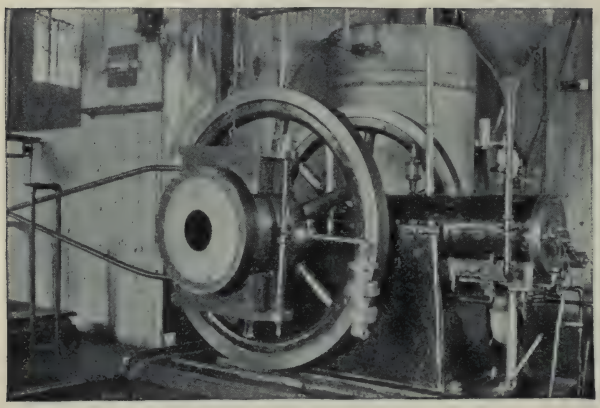


Fig. 98.

597. What would be the indicated horse-power of the gas engine, shown on page 17, and which has a piston 12 inches in diameter and a crank 8 inches long? The engine works at 150 revolutions a minute, there is an explosion every 2 revolutions, and the mean effective pressure in the cylinder is 62 pounds per square inch.

598. The speed of the governor shaft AB is 500 revolutions per minute. The 10-pound ball is to be replaced by two 20-pound balls that revolve in

planes distant 1 foot and 4 feet from the plane of the 10-pound ball. Take the distance AC as 7 inches and find R and R_1, the distances at which the 20-pound balls will revolve from the governor shaft when their centrifugal forces have the same moment about the speed controller at A as the 10-pound one alone.

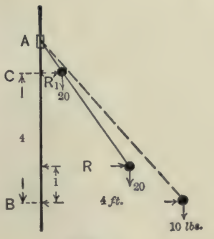


Fig. 99.

599. In the preceding problem could the distance AC be changed and still have the moments of the two 20-pound balls balance the 10? If so what is one such distance?

600. A rope manufacturer's catalogue states:

"The breaking strength of rope may be taken as 7 000 × diameter squared. For a constant transmission the best results are obtained when the tension on the driving side of the rope is not more than $\frac{1}{3}\frac{1}{5}$ of the breaking strength; and the tension on the driving side is usually twice the tension on the slack side."

Find the horse-power of a 2-inch diameter rope, of weight per foot 0.34 × diameter squared, that runs at 3 000 feet per minute when centrifugal force is considered.

Observe that the tension on slack side = centrifugal force of belt + $\frac{1}{2}$ of difference in tension.

EXAMINATIONS.

MECHANICS.*

YALE UNIVERSITY, SHEFFIELD SCIENTIFIC SCHOOL.

SENIOR MECHANICAL AND MINING ENGINEERS.

March, 1905.

1. (*a*) Define five different units of force. (*b*) A balloon is ascending with a speed which is increasing at the rate of 4 feet per second in each second. Find the apparent weight of 10 pounds weighed by a spring balance in the balloon.

2. A weight of 20 pounds rests 7 feet from the edge of a smooth horizontal table 4 feet high. A string 8 feet long passes over a smooth pulley at edge of the table and connects with a 10-pound weight. If this second weight is allowed to fall, in what time will the first weight reach the edge of the table.

3. (*a*) A cord passing over a smooth pulley carries 10 pounds at one end and 54 at the other; what will be the tension in the cord? (*b*) A shopkeeper uses a balance with arms in ratio of 5 to 6. He weighs out from alternate pans what appears to be 60 pounds. How much does he gain or lose?

4. (*a*) Define a force couple. Show that a force couple cannot be replaced by a single force. (*b*) Show

* Preparatory Studies: About 20 weeks of Mechanics in a three hour a week course, and the present course of 10 weeks with three hours a week preparation.

how to find the resultant of any number of non-concurrent forces acting on a rigid body.

5. (*a*) Find the force of attraction of a homogeneous sphere on a particle within the sphere. (*b*) The mass of the sun is 300 000 times the mass of the earth, and its radius is 100 times the radius of the earth. How far will a stone fall from rest in one second at surface of sun?

6. (*a*) A uniform rod 8 feet long, weighing 18 pounds, is fastened at one end to a vertical wall by a smooth hinge. It is kept horizontal by a string 10 feet long, attached to its free end and to a point in the wall. Find the tension in the string and the pressure on the hinge. (*b*) A uniform rod AB, 20 inches long weighing 20 pounds, rests horizontally upon two pegs whose distance apart is 8 inches. How must the rod be placed so that the pressure on the pegs may be equal when weights of 40 and 60 pounds are suspended from A and B, respectively?

7. Find by the principle of virtual work the condition of equilibrium for a differential screw considering friction.

8. A uniform ladder 70 feet long is equally inclined to a vertical wall and the horizontal ground. A man weighing 224 pounds ascends the ladder, which weighs 448 pounds. How far up the ladder can the man ascend before it slips if the coefficient of friction for the wall is $\frac{1}{3}$ and for the ground $\frac{1}{2}$?

9. Find the work lost by a shaft with a truncated pivot, bearing an end thrust.

10. A belt passing around a drum has an angle of contact a and a coefficient of friction μ. Find the horse-power which can be transmitted.

11. Two rough inclined planes are placed end to end. A body of 100 pounds rests on one of the planes, which has an inclination of 60°. A string attached to this body passes over a smooth pulley at the apex of the planes and holds another body on the second plane of inclination, 30°. If coefficient of friction for each plane is $\frac{1}{2}$, find the weight of second body to just hold the first from sliding down the plane.

sine 30° = .5 cosine 30° = .86

MECHANICS.

TUFTS COLLEGE, DEPARTMENT OF ENGINEERING.

EXAMINATION AT MID-YEAR, FEB. 6, 1905.*

Answer any eight questions.

1. In tests of cast-iron fly wheels (*Eng. News*, Dec. 15, 1904) record is given of one as follows: Diameter of wheel 4 feet, stress in each arm due to the centrifugal force of its portion of the rim 1680 pounds, weight of same portion of rim $7\frac{1}{2}$ pounds. Find bursting speed in miles per hour.

2. A leather belt treated with dressing has coefficient of friction on an iron pulley of 0.3. The belt

* Preparatory studies: Physics lectures one year, laboratory one-half year, mechanism one-half year, and present course of half year with three class hours per week and two hours of preparation for each.

encircles 200° of a pulley 10 feet in diameter. When running at 140 revolutions per minute the belt must transmit 300 horse-power. How wide should belt be if it is designed to stand 100 pounds per inch of width?

3. A wall derrick has a vertical post 9 feet high, at top a horizontal member 15 feet long, and 3 feet back from the load of 10 tons at outward end is a brace 13 feet long connecting with the vertical post at a point 4 feet up from ground. Find stresses.

4. A highway bridge 80 feet long has supports 2 feet from one end and 10 feet from the other. Uniform load on bridge is 300 pounds per linear foot. A road roller of 10 tons weight is half-way across; what load is then on each abutment?

5. A large type of locomotive recently put in service on the N. Y. C. & H. R. R. has developed approximately 2 000 horse-power. How heavy a train could this locomotive draw, at speed of 40 miles an hour, up a 2 per cent grade — (*a*) without wind or frictional resistances, (*b*) with resistances of 20 pounds per ton acting?

6. A train of 400 tons starts from a station and on a level track attains a speed of 40 miles an hour in one minute. Neglecting resistances, what would be the draw-bar pull?

7. A stiff-leg steel derrick with vertical mast 55 feet high, boom 85 feet long, set with tackle

40 feet long is raising two boilers of 50 tons total weight. Find stresses in boom and tackle and in back stay which makes an angle of 30° with vertical. If mast be made of two members joined at top and 20 feet apart at bottom what stresses must they sustain?

8. What would be the total horse-power of pumps working 12 hours per day to supply the City of Medford, 21 600 population, with 100 gallons of water per day (for each person) and forced against 60 pounds pressure (equals a height of 138 feet)? The efficiency of engines and pumps is to be 80 per cent.

9. A shell can be fired with velocity of 2 000 feet per second; neglecting resistances, how near to shore can a man-of-war be in order to have its shells just clear a fortification wall 500 feet above sea level, angle of projection being 30°?

10. Derive the formula for centrifugal force. The 20th Century express attains a speed of 60 miles per hour. When rounding a curve of 4 000 feet radius how much should the outer rail be elevated to avoid lateral pressure? (Center to center of rails is 4 feet $10\frac{3}{4}$ inches.)

11. Define acceleration, work, moment of a force, coefficient of friction. Find the least force necessary to pull a packing case of 300 pounds weight along a horizontal floor. Coefficient of friction 0.58.

Total number of problems taken during the half-year has been about 195.

MECHANICS.

TUFTS COLLEGE, DEPARTMENT OF ENGINEERING.

EXAMINATION AT MID-YEAR, FEB. 1, 1906.*

Division *a* answer any 8 questions. Division *b* answer No. 11 and 7 others.

1. In a direct-acting steam engine the piston pressure is 22 500 pounds; the connecting-rod makes a maximum angle of 15° with the line of action of the piston. Find the pressure on the guides.

2. An iron wedge having faces of equal taper that make an angle of 10° is being forced under an iron column which is supporting a load of 5 tons. The coefficient of friction for the iron surfaces is 0.18. What force is needed to push the wedge forward?

3. An electric car that is filled with passengers and weighs 25 tons goes up a grade of 1 in 100 at the speed of 10 miles an hour. The total resistances to traction are 30 pounds per ton. What horse-power must be supplied when the efficiency of the mechanism is 60 per cent? For an electro motive force of 500 volts what amperes would be necessary?

4. A shaper head that weighs 500 pounds makes its forward stroke of 12 inches in 6 seconds. The resistances of cutting and of machinery are equivalent to a coefficient of friction of 0.5. At what rate is work being done?

* Preparatory studies same as for examination of 1905 and given at bottom of page 176.

5. Coal is hoisted from a barge to a tower where it is run into a car that goes down a grade 294 feet long in 24 seconds. It strikes a cross-bar or "stopper" which is pushed back a distance of 30 feet while the car empties and for an instant comes to rest. The weight of the car is 2 000 pounds and of the coal 4 000. If the car empties uniformly during the 30 feet what is the average force of resistance that the cross-bar exerts?

6. A highway bridge of span 48 feet, width 40 feet, has two queen-post trusses of depth 9.2 feet; and each truss is divided by two posts into three equal parts. The bridge is crowded with people making a load of 150 pounds per square foot, and also an electric car one-third the way across the bridge causes an additional load equivalent to a concentrated load of 20 tons. Find the stresses in chords and posts.

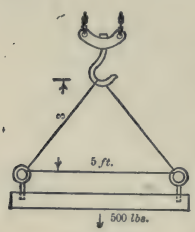


Fig. 100.

7. The head plate of a Buckeye engine is to be hoisted by a continuous rope that passes through eye bolts that are 5 feet apart, and through a chain-hoist hook that is 3 feet above the plane of the eye bolts. The rope is free to slip, and the plate weighs 500 pounds. Find the total pull that tends to break the eye bolts.

8. The center of a steel crank-pin that weighs 16 pounds is 12 inches from the center of the engine

shaft. The shaft makes 190 revolutions per minute. Find the centrifugal force caused by the pin.

9. The San Mateo Dam in California was designed for a height of 170 feet, width at top 25 feet, at base 176 feet, with a uniform batter on the water side 4 to 1, and on the back side near the top $2\frac{1}{3}$ to 1, then a curve of radius 258 feet to near the bottom where the batter is 1 to 1. The material throughout is concrete of weight 150 pounds per cubic foot. Compute approximately the factor of safety of such a section against overturning.

10. Define moment of a force and illustrate by an example. Also define and illustrate "resolve parallel and perpendicular to plane," a couple and three other important terms or equations of Mechanics. Show how to find the least force necessary to pull a box along a horizontal floor.

11.
$$\frac{W}{g}u + \frac{W'}{g}u' = \frac{W}{g}v + \frac{W'}{g}v'$$
$$v - v' = e\,(u' - u)$$

Tell what the above formulas mean.

"Horizontal range $= u \cos a \times \dfrac{2\,u \sin a}{g}$".

How obtained?

A bullet is fired with a velocity of 1 000 feet per second. What must be the angle of inclination in order that it may strike a point in the same horizontal plane at a distance of 15 625 feet?

Total number of problems taken during the half-year, about 180.

STATICS

HARVARD UNIVERSITY

First Course in Mechanics

1. Find the components of a force of 500 pounds along lines inclined to it by (*a*) 0°; (*b*) 24°; (*c*) 30°. Algebraically only.

2. Find the moment of (300 pounds 48° (− 4, 6)) about (*a*) (4, 6); (*b*) (0, 0); (*c*) (− 4, 6); (*d*) (3, − 7). Algebraically only.

3. A uniform body in the shape of an isosceles triangle with base of 60 feet and altitude of 20 feet weighs 200 pounds. It is supported at points in its base 20 feet and 60 feet respectively from the left end. Forces of 20 pounds and 40 pounds act vertically upward and downward respectively from points bisecting the left and right sloping sides respectively.

Determine the pressures upon the supports.

4. A rectangle, 10 inches by 8 inches, has one corner at the origin, two sides coincident with OX and OY, and a corner at (10, 8). Two forces, of 20 pounds each, act one along the upper edge of it toward the right, the other along the lower edge toward the left. Two more forces, of 40 pounds each, act respectively upward along left edge and downward along the right edge.

(*a*) Is the body subject either to translation or rotation?

(*b*) If any further forces be needed to cause equilibrium state the value of the simplest system that will do it.

5. Find the center of gravity of a plane figure of five sides with corners at (0, 0), (5, 0), (4, 5), (4, 3), (14, 3), (8, 0).

Solve both algebraically, and graphically, using in the latter case the general string polygon method.

6. A sphere weighing 1000 pounds rests between two smooth planes which are inclined to each other by 30°, the less steep of which is inclined 10° to the horizontal.

Determine the pressure on each plane algebraically.

7. A plane rectangular frame 60 feet high and 10 feet wide stands on two supports, one at each of the lower corners. A horizontal wind force of 4000 pounds is applied at 30 feet from the ground and a load of 6000 pounds rests at the *middle* of the top.

If the thrust of the wind be assumed to be resisted equally by the supports, determine the remaining forces at the supports.

8. Determine graphically stresses in all of the bars of the given truss. Show numerical results upon large free-hand sketch of truss.

9. Determine algebraically stresses in Q, R, and S of truss of last question without finding other stresses.

10. Determine the reactions (H_1, H_2, V_1, V_2) at the supports of the given three-hinged arch.

α	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$	α	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$
0	0.00	1.00	0.00	48	0.74	0.67	1.11
10	0.17	0.88	0.10	50	0.77	0.64	1.16
24	0.41	0.91	0.45	60	0.87	0.50	1.73
30	0.50	0.89	0.58	80	0.98	0.17	5.67
40	0.64	0.77	0.84				
45	0.71	0.71	1.00	90	1.00	0.00	∞

ANSWERS TO PROBLEMS.

In preparing this new edition two opposing suggestions have been offered to me: one that I should give all the answers to the problems, the other that I should give none. I have taken the middle ground, and am giving about half of the answers, believing that this method will serve both for engineers in practice and others who wish to know that their results are correct, and for college classes where it is often preferred that some of the answers be omitted lest the student place too much dependence on them. It is generally agreed, I think, that with students the advice frequently given by Professor Merriman in his excellent text books should be emphasized, namely: that the answers are not the main part of a problem. In fact, the student is urged not to consult the answer at the beginning of a problem, and then aim merely to get that numerical result. First an understanding of the problem should be obtained, then a diagram representing the data should be drawn, and an estimate of the answer based on experience should be noted.

Furthermore, in my own classes I require that the solutions shall be carefully made in special note books,

and that the student's method of analysis shall be plain, concise, and easily understood; for the ability to reason soundly and to demonstrate clearly should be leading aims in the study of Mechanics.

Work. (3) 3 000 foot-pounds. (6) 320 men. (10) 169 000 foot-pounds. (12) 19 800 000 foot-pounds. (15) 352 000 foot-pounds. (17) 104.8 foot-pounds. (20) 104 167 foot-pounds. (22) 20 foot-pounds. (25) 125 pounds. (28) 120 000 pounds. (32) 1.51 inches. (34) 0.54 pounds. (36) 28.5 pounds. (38) 112 pounds. (40) 522.5 pounds. (42) 6 000 foot-pounds; ratio 3 : 2. (44) 12.9 man-power. (46) $91\frac{1}{1}$ horse-power. (48) $1\frac{1}{3}$ horse-power. (50) $36\frac{4}{11}$ horse-power. (53) 25 horse-power. (55) 435 horse-power. (58) 139 kilowatts. (60) 67.8 amperes. (63) About 80 pounds per inch of width. (66) 4 inches. (68) 4 horse-power. (70) 12 miles an hour. (72) 15 pounds per ton. (74) 1 000 horse-power. (77) $\frac{89}{99}$ against friction; 23 520 000 foot-pounds wasted. (79) 10.5 horse-power theoretically. (81) 140 horse-power. (82) 107 horse-power. (83) 97.5 horse-power. (86) 1 665 horse-power. (87) 13.2 horse-power. (89) 2 566 looms. (91) 0.061. (92) 12.6 horse-power. (95) 450 horse-power. (97) 62 pounds per square inch. (100) 5.6 horse-power. (101) 1 594 horse-power. (104) 9 448 looms. (106) 5 million horse-power. (108) 0.39. (110) 157 horse-power. (113) 132 110 000 foot-pounds, or 132 million Duty. (116) 10 500 cubic feet. (118) 3.44 machines. (121) 21 hours 18 minutes. (124) 39 600 pounds. (125) 0.14 horse-power; 97 cubic inches. (127) $18\frac{3}{4}$ pounds. (129) 88 *a* R/P strokes per minute. (131) 156 tons. (133) 265 pounds. (137) 11.2 pounds per ton. (140) 1 783

amperes. (142) 393 pounds; 19.6 per cent. (145) 4.5 feet. (147) 1 856 horse-power. (149) 12 758 foot-pounds; 1 450 pounds. (152) 0.17 horse-power. (156) 549 pounds. (158) 1 250 tons. (159) 15 625 feet height. (162) 750 pounds. (164) 44.3 turns. (166) Ratio 1 to 1.94. (167) Average of 866 foot-pounds. (169) 62.5 cubic feet. (171) 15.2 horse-power. **Force.** (173) 300 pounds. (176) 10 pounds. (178) 43 units; 25. (181) 29 pounds. (183) 150 pounds; 90. (186) Rafters 6.32 tons; tie rod 6 tons. (188) Boom 77.3 tons; tackle 36.4. (191) 50 pounds. (193) 580 pounds. (194) 6 030 pounds. (196) 117.1 pounds; 82.8. (199) 2.8 pounds; 9.6. (200) $\cos^3\theta = b/a$, b being distance from C to AB and $2a$ the length of the rod. (203) Perpendicular to plane. (205) 1020 pounds; 1 000. (209) 2.45 tons. (211) 1.15 tons. (213) 86.6 pounds; 100. (217) 1 460 pounds; 1 990. (218) 77 pounds. (220) D being area of triangle, $P = W\,b\,(a^2 + c^2 - b^2) \div 4\,c\,D$; $Q = W\,a\,(b^2 + c^2 - a^2)/4\,c\,D$. (222) In guy 10.3 tons; in legs 18.6 tons. (224) Back stay, 90 tons; legs, 100. (226) 7.8 tons; 6.5. (228) Back stays, 81 tons; A-frame 36; wire rope 29; upper boom 46; lower 51; guy or tackle 49.5. (229) 1.16 tons; 0.55; 0.53. (232) 14.2 pounds. (233) 13 units at $\tan^{-1}\frac{5}{12}$ with AB. (234) $2\sqrt{2}\,P$ parallel to CA at distance from AC of $3\sqrt{2}/2 \times$ AB. (238) Ratio 1 to $\sqrt{3}$. (242) Tension $= W\,l \div 2\sqrt{l^2 - c^2 \sin\theta}$. (246) 115.5 pounds; 57.7. (252) 6 250 pounds; 5 000. (257) 6 inches from end. (259) 9 inches from middle; 18 pounds. (263) 5 inches from the middle. (266) 7 feet. (268) $106\frac{2}{3}$ pounds. (270) 3.5 pounds; $\frac{1}{2}$ inch from middle. (272) $4\frac{1}{3}$ tons; $3\frac{2}{3}$. (276) 6.5 pounds. (278) 9.5 pounds. (280) 11 inches

and 6. (282) Posts 40 tons; lower chord, 37.5 and 32.5; upper chord 32.5. (284) Post 6 tons; tie 12.8; chord 12.5. (286) 75 pounds. (287) 33.6 pounds. (290) 540. (292) On line bisecting vertical angle $\frac{2}{3}$ from vertex. (293) $2\sqrt{3}\,a/9$, $\sqrt{3}\,a/3$, $4\sqrt{3}\,a$ from the sides, if each side $= 2a$. (294) $6\sqrt{3}\,a/11$, $3\sqrt{3}\,a/11$, $2\sqrt{3}\,a/11$ from sides; outside the triangle at distance $6\sqrt{3}\,a/5$, $-3\sqrt{3}\,a/5$, $2\sqrt{3}\,a/5$. (296) Any point of line parallel to CD passing through X which is in BC produced so that $CX = 2\,BC$. (299) 5 units acting parallel to BD, cutting BC produced at X, so that $4\,CX = BC$. (302) 124 pounds, 92, 134. (306) At point 15 and 16 inches from adjacent sides. (309) $2\frac{1}{4}$ feet from rim. (313) If D be the middle point of BC, R is represented in magnitude by 2 AD, and acts through X parallel to DA, X being in DC or DB, so that $DX = BC/8$. (315) He loses 1 pound. (317) 1 inch. (318) 85.9 pounds. (321) 15 pounds each. (323) At C force is horizontal, and $= W\sqrt{3}/2$; at B $\tan^{-1}\sqrt{3}/2$ to vertical and $= W\sqrt{7}/2$. (326) Length of stick from nail to wall $\sqrt[3]{3}$: pressure $= 8\sqrt[3]{3}$ ounces and $8\sqrt{\sqrt[3]{9} - 1}$. (327) 18 900 pounds. (328) 35.3 feet. (331) P = 15 000 pounds. (332) $\frac{3}{8}$ of length from end where pressure is 4 pounds. (335) 1.33 inches. (339) 1 inch from AC, $1\frac{7}{9}$ from AB. (342) It divides the face to which the cover is hinged in ratio of 1 to 2. (345) From left-hand edge 2.84 inches; 5.36 inches. (348) $2\cos\theta = 3\cos(\pi - \theta)/3$, θ being angle with horizontal. (350) $h = r\sqrt{3}$. (352) $\frac{1}{7}$. (355) 373 pounds. (357) $1/\sqrt{3}$. (359) 200 pounds. (362) $\frac{2}{11}$. (366) $\frac{1}{4}$; inclination, $\tan^{-1}\frac{3}{4}$. (369) 433 pounds. (371) 1140 pounds; 314. (374) 60°. (376) $\mu W r/(1 + \sqrt[2]{})\sin\alpha$, r being radius, and W the weight of wheel. (378) 47

feet. (380) $1/\sqrt{3}$. (382) 100 pounds. (388) About 2.08. (393) 58 pounds. (394) 2 800 pounds. (396) 898 pounds. (398) $3\frac{1}{2}$. (400) 13 inches. (401) 37.6 inches. (405) 504 pounds. (406) 1.92 horse-power. (407) W/P = 0.95 or 1.05. (408) 137 thermal units. (410) 0.44 horse-power. (412) 3.5 horse-power. **Motion.** (415) 35 feet. (417) $13\frac{7}{11}$ miles per hour; $27\frac{3}{11}$. (419) 30 miles an hour. (422) 150 feet; 200. (424) $13\frac{3}{4}$ pounds per ton. (425) 99 feet. (426) 5 seconds. (428) 6 seconds; 112 feet per second. (431) $\sqrt{ag}$ feet per second. (433) 231 feet. (435) 4 080 feet. (436) In $\sqrt{h/2g}$ seconds, and $3h/4$ feet from ground. (438) 350 feet. (440) $u\sqrt{3}/2$; $u/2$. (442) 17.6 feet. (444) About $\frac{1}{3}$ mile up-stream. (446) 24.5 miles an hour. (449) Northwest 6 $\sqrt{2}$ miles an hour. (451) 24π. (454) 65.5 miles an hour; 0.27 miles. (457) 300 pounds. (459) 24.3 amperes for maximum velocity. (462) 1.9 miles. (464) 0.014. (467) $16\sqrt{5}$ feet per second. (469) 7.25 feet. (470) 8 feet per second. (473) $31\frac{7}{6}$ pounds. (475) 6 938 pounds. (478) (*a*) $\frac{13}{8}g$ feet; (*b*) 70 pounds, 140 and $186\frac{2}{3}$. (479) 1 second. (481) 15° or 75°. (483) $44\sqrt{2}$ feet. (486) 3903 feet inside of city. (487) 7.16 miles. (489) 8 600 feet; 31 250 000 foot-pounds. (494) 0.3 tons. (497) 2.83 pounds. (500) $\mathrm{Tan}^{-1}\frac{1}{2}\frac{2}{0}\frac{1}{0}\frac{1}{0}$. (503) 43 revolutions per minute. (506) 6.1 tons. (507) 321 tons. (510) 23.1 feet. (514) 3.4 pounds. (515) $3\ 666\frac{2}{3}$ feet. (519) 3.1 feet per second. (521) 496.8 feet per second. (525) -1 feet per second; $+2$. (527) A returns 5 feet per second; B moves at 45° with its course and velocity of $10\sqrt{2}$. **Review.** (529) (*a*) 58 tons. (531) 56.7 per cent. (534) 60 000 pounds close to tower; 47 000 in middle. (535) 52 222 pounds. (536) 17

revolutions. (538) 7.25 feet. (541) In bolt 27 500 pounds. (543) In leg 63.24 tons; in inclined members, 18.75 tons. (545) 0.036. (549) 1 250 pounds. (552) 27.8 feet per second. (557) 472 tons. (558) 160. (559) 682 revolutions per minute; 1910 amperes for the 8 motors. (560) 1 453 horse-power. (563) 0.19. (564) 16 879 pounds; 8.1 per cent. (566) 2 720 pounds per square inch. (568) $\cos\theta = P\ 2\ W$. (571) Horizontal stress 1.8 tons; vertical additional 0.9 tons. (573) 2.0 horse-power. (574) 35 pounds. (575) (*a*) 41 800 pounds; (*b*) 458 000 pounds; (*c*) 1 195 gallons. (577) 7 419 pounds. (578) 19 200 pounds. (579) 14 631 pounds, 24 769. (580) 1 780 pounds. (583) In air, boom 16.57 tons; in water, boom 14.45 tons. (586) $4\frac{1}{2}$ tons. (587) 15.2 feet. (588) 12.15 feet from large end. (589) 33.75 feet; $\frac{1}{60}$ second. (590) 9 518 pounds. (592) (*b*) 19.2 inches. (594) 54.2 pounds; 10.4. (596) 80.6 per cent. (600) 42 horse-power.

Falling Bodies: Velocity Acquired by a Body Falling a Given Height.

Height.	Velocity.	Height.	Velocity.	Height.	Velocity.	Height.	Velocity.	Height.	Velocity.	Height.	Velocity.
feet.	feet p.sec.	feet.	feet p.sec.	feet.	feet p.sec.	feet.	feet p.sec.	feet.	feet p.sec.	feet.	feet p.sec.
.005	.57	.39	5.01	1.20	8.79	5.	17.9	23.	38.5	72	68.1
.010	.80	.40	5.07	1.22	8.87	.2	18.3	.5	38.9	73	68.5
.015	.98	.41	5.14	1.24	8.94	.4	18.7	24.	39.3	74	69.0
.020	1.13	.42	5.20	1.26	9.01	.6	19.0	.5	39.7	75	69.5
.025	1.27	.43	5.26	1.28	9.08	.8	19.3	25	40.1	76	69.9
.030	1.39	.44	5.32	1.30	9.15	6.	19.7	26	40.9	77	70.4
.035	1.50	.45	5.38	1.32	9.21	.2	20.0	27	41.7	78	70.9
.040	1.60	.46	5.44	1.34	9.29	.4	20.3	28	42.5	79	71.3
.045	1.70	.47	5.50	1.36	9.36	.6	20.6	29	43.2	80	71.8
.050	1.79	.48	5.56	1.38	9.43	.8	20.9	30	43.9	81	72.2
.055	1.88	.49	5.61	1.40	9.49	7.	21.2	31	44.7	82	72.6
.060	1.97	.50	5.67	1.42	9.57	.2	21.5	32	45.4	83	73.1
.065	2.04	.51	5.73	1.44	9.62	.4	21.8	33	46.1	84	73.5
.070	2.12	.52	5.78	1.46	9.70	.6	22.1	34	46.8	85	74.0
.075	2.20	.53	5.84	1.48	9.77	.8	22.4	35	47.4	86	74.4
.080	2.27	.54	5.90	1.50	9.82	8.	22.7	36	48.1	87	74.8
.085	2.34	.55	5.95	1.52	9.90	.2	23.0	37	48.8	88	75.3
.090	2.41	.56	6.00	1.54	9.96	.4	23.3	38	49.4	89	75.7
.095	2.47	.57	6.06	1.56	10.0	.6	23.5	39	50.1	90	76.1
.100	2.54	.58	6.11	1.58	10.1	.8	23.8	40	50.7	91	76.5
.105	2.60	.59	6.16	1.60	10.2	9.	24.1	41	51.4	92	76.9
.110	2.66	.60	6.21	1.65	10.3	.2	24.3	42	52.0	93	77.4
.115	2.72	.62	6.32	1.70	10.5	.4	24.6	43	52.6	94	77.8
.120	2.78	.64	6.42	1.75	10.6	.6	24.8	44	53.2	95	78.2
.125	2.84	.66	6.52	1.80	10.8	.8	25.1	45	53.8	96	78.6
.130	2.89	.68	6.61	1.90	11.1	10.	25.4	46	54.4	97	79.0
.14	3.00	.70	6.71	2.	11.4	.5	26.0	47	55.0	98	79.4
.15	3.11	.72	6.81	2.1	11.7	11.	26.6	48	55.6	99	79.8
.16	3.21	.74	6.90	2.2	11.9	.5	27.2	49	56.1	100	80.2
.17	3.31	.76	6.99	2.3	12.2	12.	27.8	50	56.7	125	89.7
.18	3.40	.78	7.09	2.4	12.4	.5	28.4	51	57.3	150	98.3
.19	3.50	.80	7.18	2.5	12.6	13.	28.9	52	57.8	175	106
.20	3.59	.82	7.26	2.6	12.9	.5	29.5	53	58.4	200	114
.21	3.68	.84	7.35	2.7	13.2	14.	30.0	54	59.0	225	120
.22	3.76	.86	7.44	2.8	13.4	.5	30.5	55	59.5	250	126
.23	3.85	.88	7.53	2.9	13.7	15.	31.1	56	60.0	275	133
.24	3.93	.90	7.61	3.	13.9	.5	31.6	57	60.6	300	139
.25	4.01	.92	7.69	3.1	14.1	16.	32.1	58	61.1	350	150
.26	4.09	.94	7.78	3.2	14.3	.5	32.6	59	61.6	400	160
.27	4.17	.96	7.86	3.3	14.5	17.	33.1	60	62.1	450	170
.28	4.25	.98	7.94	3.4	14.8	.5	33.6	61	62.7	500	179
.29	4.32	1.00	8.02	3.5	15.0	18.	34.0	62	63.2	550	188
.30	4.39	1.02	8.10	3.6	15.2	.5	34.5	63	63.7	600	197
.31	4.47	1.04	8.18	3.7	15.4	19.	35.0	64	64.2	700	212
.32	4.54	1.06	8.26	3.8	15.6	.5	35.4	65	64.7	800	227
.33	4.61	1.08	8.34	3.9	15.8	20.	35.9	66	65.2	900	241
.34	4.68	1.10	8.41	4.	16.0	.5	36.3	67	65.7	1000	254
.35	4.74	1.12	8.49	.2	16.4	21.	36.8	68	66.1	2000	359
.36	4.81	1.14	8.57	.4	16.8	.5	37.2	69	66.6	3000	439
.37	4.88	1.16	8.64	.6	17.2	22.	37.6	70	67.1	4000	507
.38	4.94	1.18	8.72	.8	17.6	.5	38.1	71	67.6	5000	567

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Angle	Sin	Tan	Sec	Cosec	Cot	Cos	
0	0.	0.	.0	∞	∞	1.	90
1	0.0175	0.0175	1.0001	57.299	57.290	0.9998	89
2	.0349	.0349	1.0006	28.654	28.636	.9994	88
3	.0523	.0524	1.0014	19.107	19.081	.9986	87
4	.0698	.0699	1.0024	14.336	14.301	.9976	86
5	.0872	.0875	1.0038	11.474	11.430	.9962	85
6	0.1045	0.1051	1.0055	9.5668	9.5144	0.9945	84
7	.1219	.1228	1.0075	8.2055	8.1443	.9925	83
8	.1392	.1405	1.0098	7.1853	7.1154	.9903	82
9	.1564	.1584	1.0125	6.3925	6.3138	.9877	81
10	.1736	.1763	1.0154	5.7588	5.6713	.9848	80
11	0.1908	0.1944	1.0187	5 2408	5.1446	0.9816	79
12	.2079	.2126	1.0223	4.8097	4.7046	.9781	78
13	.2250	.2309	1.0263	4 4454	4.3315	.9744	77
14	.2419	.2493	1.0306	4.1336	4.0108	.9703	76
15	.2588	.2679	1.0353	3.8637	3 7321	.9659	75
16	0.2756	0.2867	1.0403	3.6280	3.4874	0.9613	74
17	.2924	.3057	1.0457	3.4203	3.2709	.9563	73
18	.3090	.3249	1.0515	3 2361	3 0777	.9511	72
19	.3256	.3443	1.0576	3.0716	2.9042	.9455	71
20	.3420	.3640	1.0642	2.9238	2.7475	.9397	70
21	0.3584	0.3839	1.0712	2.7904	2.6051	0.9336	69
22	.3746	.4040	1.0785	2.6695	2.4751	.9272	68
23	.3907	.4245	1.0864	2.5593	2.3559	.9205	67
24	.4067	.4452	1.0946	2.4586	2.2460	.9135	66
25	.4226	.4663	1.1034	2.3662	2.1445	.9063	65
26	0.4384	0.4877	1.1126	2.2812	2.0503	0.8988	64
27	.4540	.5095	1.1223	2.2027	1.9626	.8910	63
28	.4695	.5317	1.1326	2.1301	1.8807	.8829	62
29	.4848	.5543	1.1434	2.0627	1.8040	.8746	61
30	.5000	.5774	1.1547	2.0000	1.7321	.8660	60
31	0.5150	0.6009	1.1666	1.9416	1.6643	0.8572	59
32	.5299	.6249	1.1792	1.8871	1.6003	.8480	58
33	.5446	.6494	1.1924	1.8361	1.5399	.8387	57
34	.5592	.6745	1.2062	1.7883	1.4826	.8290	56
35	.5736	.7002	1.2208	1.7435	1.4281	.8192	55
36	0.5878	0.7265	1.2361	1.7013	1.3764	0.8090	54
37	.6018	.7536	1.2521	1.6616	1.3270	.7986	53
38	.6157	.7813	1.2690	1.6243	1.2799	.7880	52
39	.6293	.8098	1.2868	1.5890	1.2349	.7771	51
40	.6428	.8391	1.3054	1.5557	1.1918	.7660	50
41	0.6561	0.8693	1.3250	1.5243	1.1504	0.7547	49
42	.6691	.9004	1.3456	1.4945	1.1106	.7431	48
43	.6820	.9325	1.3673	1.4663	1.0724	.7314	47
44	.6947	.9657	1.3902	1.4396	1.0355	.7193	46
45	.7071	1.	1.4142	1.4142	1.	.7071	45
	Cos	Cot	Cosec	Sec	Tan	Sin	Angle

A FEW IMPORTANT UNIT VALUES TO BE USED IN SOLVING THESE PROBLEMS

30 miles an hour	= 44 feet per second
1 ton	= 2 000 pounds
1 fathom	= 6 feet
1 knot	= 6 080 feet
1 cubic foot of water	= $62\frac{1}{2}$ pounds
	= $7\frac{1}{2}$ gallons
1 gallon of water	= $8\frac{1}{3}$ pounds
1 pound of water pressure	= 2.304 feet head
1 British thermal unit	= 778 foot-pounds of energy
g, acceleration of gravity	= 32 feet per second per second, unless otherwise specified
e the base of Napierian system of logarithms	= 2.7 1828 1828
1 horse-power	= 746 watts
1 kilowatt	= 1.34 horse-power

Watts = volts × amperes

IMPORTANT FUNCTIONS OF ANGLES AND TRIGONOMETRIC RELATIONS.

	0°	30°	45°	60°	90	120°
Sine	0	$\frac{1}{2}$ .500	$\frac{1}{\sqrt{2}}$ .707	$\frac{\sqrt{3}}{2}$ .866	1	$\frac{\sqrt{3}}{2}$ .866
Cosine	1	$\frac{\sqrt{3}}{2}$ .866	$\frac{1}{\sqrt{2}}$ .707	$\frac{1}{2}$ .500	0	$-\frac{1}{2}$ $-$ .500
Tangent	0	$\frac{1}{\sqrt{3}}$ .577	1	$\sqrt{3}$ 1.732	Infinite	$-\sqrt{3}$ $-$ 1.732

$\text{Sin} = \frac{\text{Perp}}{\text{Hypot}}$ $\quad \text{Cos} = \frac{\text{Base}}{\text{Hypot}}$ $\quad \text{Tan} = \frac{\text{Perp}}{\text{Base}}$

$\text{Tan} = \frac{\text{Sin}}{\text{Cos}}$ $\quad \text{Cot} = \frac{1}{\text{Tan}}$ $\quad \text{Sec} = \frac{1}{\text{Cos}}$

$\text{Cosec} = \frac{1}{\text{Sin}}$ $\quad \text{Vers} = 1 - \cos$ $\quad a : b = \text{Sin } A : \text{Sin } B$

$\text{Sin}(A + B) = \text{Sin } A \text{ Cos } B + \text{Cos } A \text{ Sin } B$ $\quad c = \sqrt{a^2 + b^2 - 2ab \text{ Cos } C}$

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